Department of Physics & Astronomy
Second Year Laboratory

Courses: 2B30, 2B40, 2B41 & 2B42

Experiment E4

Study of Thermal Noise in a Resistor

Relevant Lecture Courses

1B28	1st year Thermal Physics
PHAS 2201	Electricity and Magnetism
2B65	Space Science, instrumentation and techniques

This document can be found on the Departmental Web site

Experimental Objectives and Planning:

It is important that you understand certain new elements embodied in this experiment. As you proceed with the experiment consider the following items and write very brief notes answering each question below.

1. This experiment is composed of three sections. *What is the aim of each and how does each part of the experiment support the other two?*

2. In parts 1 and 2 of this project you will be investigating the dependence of the noise on the resistance and temperature. *From your reading of equation 4 below what dependencies do you expect? How will you verify this? Finally how will you estimate any errors required for a complete analysis?*

3. In part three of the experiment you will investigate how the gain of an amplifier varies with frequency. *Why is this important to your study?* You will be using a sinusoidal signal to make this investigation. *Why can’t you use a square wave signal for this part of the project? Finally how might you estimate an error in the equivalent noise bandwidth?*

4. In your calculation of Boltzmann’s constant you may use the method of weighted mean to give a “best value”. *At what point should you use this method of analysis? Explain your choice and what errors you would use for this.*

5. In the last experiment you will be using a device called an attenuator. This is essentially a potential divider. *What does an attenuator do, why is it necessary to use such a device and explain the principle of its operation?*

After reading the script and before beginning the experiment have a short discussion with you lab partner and produce a brief summary of the steps you need to take to complete the experiment.

1 BACKGROUND:

At any temperature above absolute zero the thermal motion of charge carriers within a conducting medium, such as a resistor, will induce a randomly fluctuating voltage across its external terminals. This effect is usually known as resistor or Johnson noise (Johnson being the first person to demonstrate the effect experimentally). It was shown by Nyquist (1928) in a derivation based on classical statistical mechanics that the magnitude of the voltage noise is given by equation 1:

$$\overline{e}_n^{\,2} = 4kRT\Delta f \qquad (1)$$

where $\overline{\mathbf{e}}_{\mathbf{n}}^{\,2}$ is the mean square voltage induced (within a frequency range f to $f + \Delta f$) across a resistor R at temperature T, k being Boltzmann's constant.

In practice, e_n is small. For example, if R=10 kΩ, T=293K and f=10 kHz, then e_n~1μV.

Even so this random fluctuation limits the accuracy and sensitivity of electronic measurements in many cases. In this experiment the noise generated by a warm resistor is amplified electronically to a level where it is easily measurable with a standard commercial r.m.s. voltmeter. The way in which the noise voltage varies as a function of R and T is investigated and a value for k derived from the experimental measurements.

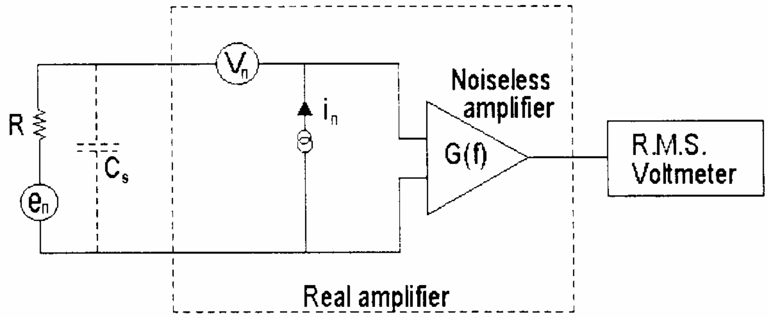

Figure 1

The experimental arrangement of resistor, amplifier and voltmeter may be represented diagrammatically as in Fig. 1.

As shown, the noisy resistor can be represented as a noiseless resistor of equivalent value in series with a voltage source e_n, given by equation (1). The introduction of an amplifier between the resistor and the voltmeter introduces the following factors into the analysis.

1. All amplifiers introduce additional noise into the amplified signal. In a well designed amplifier this noise originates mainly from the input stage and it is reasonable to consider the noisy amplifier as an equivalent noiseless amplifier in conjunction with a series voltage noise source v_n and a parallel current noise source i_n at its input as shown in Fig. 1. Since both v_n and i_n are in random form and uncorrelated with each other and with e_n the resistor noise, the mean square noises add linearly and the total equivalent input noise is given by

$$V_{in}^2 = e_n^2 + V_n^2 + (i_n R)^2 \qquad (2)$$

For the amplifier used in this experiment the current noise i_n can be neglected. The voltage noise v_n can be determined by making $e_n = 0$, i.e. by shorting the amplifier input to make $R = 0$.

2. The amplifier gain is a function of frequency and the noise bandwidths will be dependent on the total frequency response of the amplifier and voltmeter. For frequencies between, f and $f + df$, the contribution to the mean square noise voltage as measured on the voltmeter will be

$$d(V_n^2(R)) = 4kTRdf\,xG^2(f) \qquad (3)$$

where $G(f)$ is the voltage gain of the amplifier/meter combination at frequency f. Over all frequencies, this will sum to

$$V_n^2(R) = 4kTR\int_0^\infty G^2(f)df \qquad (4)$$

The quantity $\int_0^\infty G^2(f)df$ is known as the equivalent noise band width of the system. It can be determined experimentally by measuring $G(f)$ as a function of f, plotting $G^2(f)$ against f and integrating the area under this curve.

2 EXPERIMENTAL ARRANGEMENT:

The experimental arrangement for observing and measuring resistor noise is shown in Figure 2. The resistance box contains a range of different valued resistors individually selectable by switching. The amplifier is a low noise type with a maximum gain of around 5 x 10^4 and a bandwidth of ~10^4Hz. The oscilloscope is not used for any measurements but provides an extremely useful visual display of the random noise waveform. There are a number of practical points which have to be considered in the experiment design.

Apparatus List:

switched resistor box
oscilloscope
digital voltmeter
function generator
low noise amplifier
r.m.s. voltmeter
attenuator
resistive probe

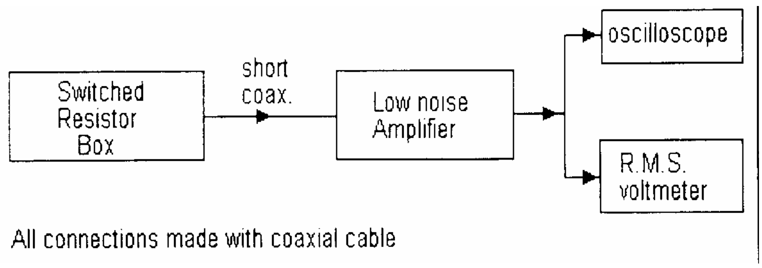

Figure 2

Note, in particular, that:

1 Although in principle the Johnson noise is not dependent on the resistor material, some types of resistor (notably carbon film) produce excess noise. Resistors of the thin metal film type are therefore used here.

2. The equivalent circuit of the amplifier input should include a stray capacity (due to leads, input capacity etc) as shown dotted in Fig. 1. Together with the input resistance R, this forms a low pass filter on the input noise voltage with a cut off frequency ~$1/2\pi RC_s$. To avoid the system bandwidth changing with R it is necessary to restrict the maximum value of R so that $1/2\pi RC_s$ is always greater than the maximum frequencies to

which the amplifier/meter responds. Clearly it helps to keep C_s to a minimum, which means connections between the resistance box and the amplifier input should be kept short. In the experiment as designed, C_s~5 x 10^{-11} F and R_{max} ~ 25kΩ so that $1/2\pi RC_s$ ~130kHz min. This is some way above the system frequency response.

3. It is important to appreciate that the meter used to measure the amplified noise voltage must be of the true r.m.s. reading type. Many meters read an average of the rectified voltage and are calibrated for sinusoidal waveforms only. Thermal noise in a resistor is readily seen from the oscilloscope to be nothing like a sinusoid. The use of such a meter to measure spiky waveforms such as random noise can lead to significant errors.

4. Small voltages and high impedance levels at the amplifier input make the system very susceptible to capacitive pick up from stray AC fields in the laboratory. To minimise this possibility the amplifier and resistance box are shielded by earthed metal cases and connections are made using coaxial cables.

The experimental arrangement for measuring the frequency response of the amplifier/r.m.s. voltmeter is shown in Fig.3. In principle all that is needed to determine the response at a given frequency is to apply a sine wave of known amplitude to the amplifier input and read the output on the r.m.s. voltmeter. In practice the amplifier gain is high and input signals of only milliVolts are allowable. Such small signal amplitudes are difficult to measure accurately. Accordingly, the input signal is generated and measured at a much higher level (~2V) where it can be measured using a standard digital voltmeter (d.v.m.) (a true r.m.s. voltmeter is unnecessary since the signal now being measured is sinusoidal). The high level signal is then attenuated in a resistive network (Fig.5) to reduce it to a level suitable for the amplifier input. The actual amplifier input can then be derived from the measured input voltage and the attenuation calculated from the resistance values in the dividing network.

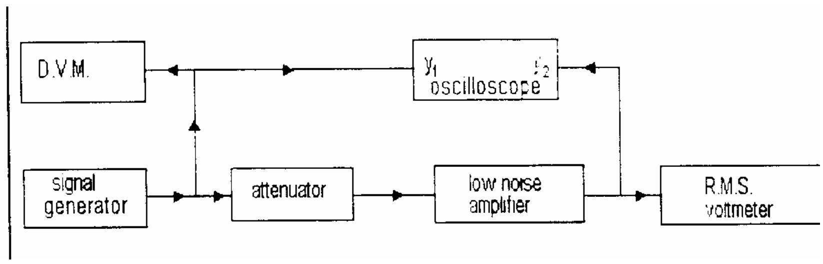

Figure 3. Bandwidth calibration

3 EXPERIMENTS:

3.1 Variation of resistor noise with resistance *R*

Set up the apparatus shown in Fig.2. Look at the noise voltage displayed on the oscilloscope and observe its spiky random nature. Note how the amplitude increases with the value of the input resistance selected.

Presumably some of the signal could arise from interference picked up from the 50Hz mains electricity supplies. However you can see whether or not this is the case. With the time base of the oscilloscope set to l0 msec/cm make sure that there is no obvious 50Hz interference being picked up by the amplifier. If there is, it will appear as a sinusoidal envelope to the spiky noise signal. If all appears satisfactory, short the amplifier input (set R = 0) and read the output voltage on the r.m.s. voltmeter. (**Note:** To set up the voltmeter as a r.m.s. meter, set the dial to Vdc or mVdc and depress the yellow select switch.) This represents the noise due to the amplifier alone.

Next, select one of the resistors and read the output voltage again. This will include contributions from both resistor and amplifier noise. The amplifier noise can be subtracted using the relation

$$V_0^2(R) = V_0^2(R + A) - V_0^2(A)$$

where $V_O(R)$, $V_O(R + A)$ and $V_O(A)$ are the output voltages associated with the resistor noise, the combined resistor and amplifier noise, and the amplifier noise respectively.

Repeat the observations for each of the resistor values supplied. Plot V_O^2 against R. According to equation (4) one would expect this to be a straight line with gradient equal to $4kT'\int_0^\infty G^2(f)df$ where T' is the laboratory temperature.

3.2 Variation of resistor noise with temperature *T*:

Replace the resistance box with the resistance probe (shown in the cross section in Fig.4). This contains a 10kΩ resistance (1% tolerance) mounted near the tip of the thin stainless steel tube. Immerse the active end of the probe in ice/water (273K) and note the output reading. Correct this to remove amplifier noise as in experiment 3.1.

Repeat with the probe immersed in as many of the following as possible:

a) Near boiling water 373K
b) Dry ice (solid carbon dioxide) 195K
c) Liquid nitrogen 77K
d) Room temperature

(A digital thermometer can be obtained from the Lab technician to measure the actual temperatures of the substances)

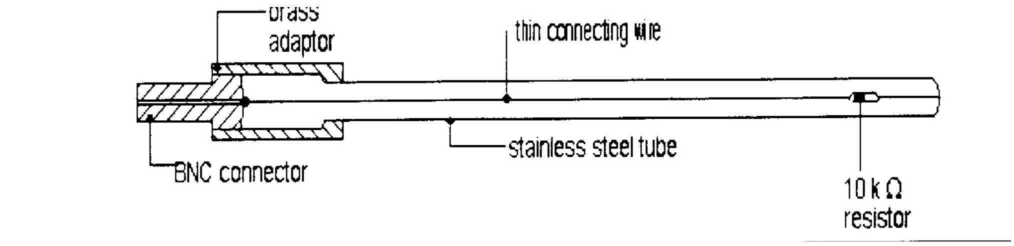

Figure 4. Resistive probe

Plot the corrected output noise (Voltage)^2 against *T*. According to equation (4), one would expect this graph to be a straight line passing through the origin with a gradient $4kR'\int_0^\infty G^2(f)df$ where *R'* is the resistance of the probe.

You may make a quick check that your data so far is internally consistent in that you would expect the ratio of the slopes of the graph of $(V_{noise})^2$ against *R*, and that of $(V_{noise})^2$ against *T*, to be in the ratio *T'/R'* , i.e. room temperature divided by probe resistance.

3.3 Determination of Boltzmann's Constant *k*:

The measurements of experiments 3.1 and 3.2 give values for the quantities $4kT'\int_0^\infty G^2(f)df$ and $4kR'\int_0^\infty G^2(f)df$ respectively, together with uncertainties, obtained from the slopes of the two graphs concerned.

To determine the equivalent noise bandwidth $\int_0^\infty G^2(f)df$, the gain *G*(*f*) of the amplifier must be measured as a function of frequency. Set up the apparatus of Fig.3 and set the signal generator to produce sine waves at about 5kHz. Then adjust the input amplitude V_{in} produced by the signal generator until the r.m.s. output voltmeter, V_0, reads around 2V. Note this reading of V_0 and correct it for amplifier noise as in the previous experiments. Note also the input voltage V_{in}.

Then repeat measurements of V_0 and V_{in} varying the frequency of the signal generator over the frequency range for which the amplifier has a significant, i.e. non-zero gain (a range of about 200Hz up to 50kHz should be suitable). A knowledge of the constant attenuation *A*, (*A*<<1), then enables *G*(*f*), (*G*(*f*)>>1), to be calculated for each frequency because the gain at that frequency is then given by

$$G = \frac{V_0}{V_{in}} A^{-1}$$

where V_O is the corrected output voltage.

Be careful when using the signal generator that, not only should the output be set to sinusoidal, but that the left hand grey knob (marked “cal”) should be rotated fully anti-clockwise, otherwise the output will not be sinusoidal (check on ‘scope).

The attenuation, A**,** can is calculated from the divider network values shown in Fig.5.

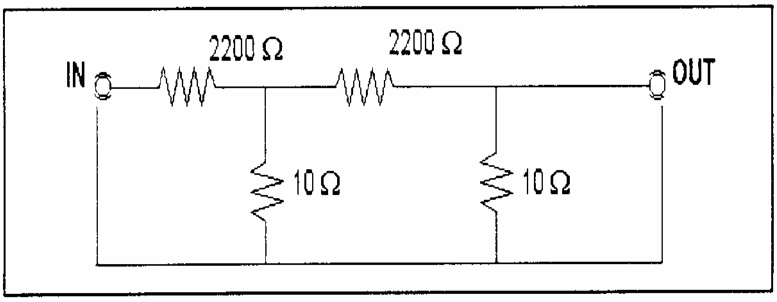

Figure 5 Resistive attenuator

To calculate the attenuation, A, note that the above circuit may be redrawn as shown in Figure 6 and note that the resistance of the series/parallel combination contained within the circle can be easily calculated, and hence the potential at point V' deduced in terms of V_{in} . Once V' is known in terms of V_{in} then so is V_{out}, and hence A. (N.B. $A= V_{out} / V_{in}$ with reference to the attenuator circuit.) Avoid making any approximations (e.g. 2200 >> 10) in your calculation of A.

Make a rough estimate of the uncertainty on A, given that 1% tolerance resistors were used in its construction. Be aware that resistors are normally purchased not as "exact" values, but are merely guaranteed to lie within a given tolerance band. The resistors used in the construction of the attenuators have their nominal value ±1%, thus your calculated value of A has an uncertainty. (Do not compute ΔA exactly, but derive a suitable approximation from your analytic expression for A.)

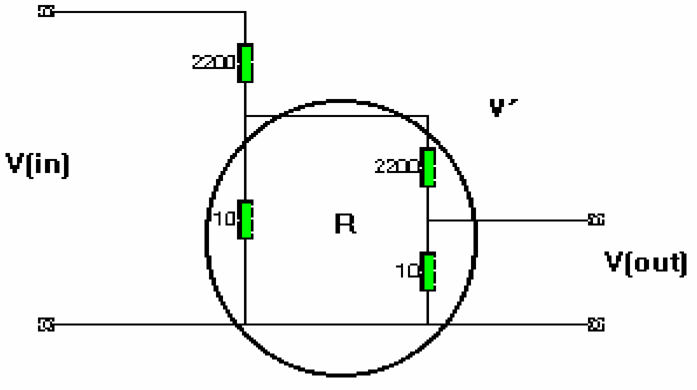

Figure 6 Resistive attenuator

Having calculated A, you can then plot $G^2(f)$ against f and find the area under the curve to obtain the equivalent noise bandwidth, and the associated uncertainty. Use this and the results from experiments 3.1 and 3.2 to calculate two values of k, and their uncertainties.

Carefully combine the two sets of data via a weighted mean calculation to evaluate ‘your best’ value of *k* and compare your results with the accepted value for *k*.

4 REFERENCES:

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Update

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