Course and assignment number:

Tutor Marked Assignment

S357 02

Make sure you know how to complete and send in your TMA and PT3 form: detailed instructions are given in your student handbook (or supplement).

Covering: **Units 4–7**

Cut-off date:
Friday 6 June 1997

length should be invariant

This assignment consists of three questions respectively weighted at 45%, 20% and 35% of the total marks for the assignment.

Question 1

This question carries 45% of the marks for this assignment.

A thin straight rod of very great length, L, lies stationary in an inertial reference frame S′ from $\mathbf{x}' = (-L/2, 0, 0)$ to $\mathbf{x}' = (L/2, 0, 0)$ and carries positive charge, uniformly distributed along its length. A positive test charge of small magnitude, q, is located permanently at the point $\mathbf{x}' = (0, d, 0)$ where $d \ll L$. The origin of S′ moves with constant speed V along the positive 1-axis of a second reference frame S, which has all its axes parallel to those of S′. It may be assumed that all the observations referred to below, whether in S or S′, are made at points close to the rod and very far from its ends.

(i) Explain briefly why you would expect the magnitudes, $|\mathbf{E}|$ and $|\mathbf{B}|$, of the electric and magnetic fields observed at a point $\mathbf{x}$ in S to depend only on r where

$$r = \sqrt{(x^2)^2 + (x^3)^2}$$

and not on any other combination of the position coordinates.

(ii) Make sketches of the fields $\mathbf{E}$ and $\mathbf{B}$ observed in S, indicating clearly the directions of the fields.

(iii) Given that

$$|\mathbf{E}| = \frac{\lambda}{2\pi\epsilon_0 r}$$

and

$$|\mathbf{B}| = \frac{\mu_0 \lambda V}{2\pi r}$$ should not be velocity dep

where λ is the linear charge density on the rod in $\mathrm{C\,m^{-1}}$, calculate the force, $\mathbf{F}$, exerted on the test charge by the fields in S. (Show your working.)

(iv) Write down an expression for the linear density of charge on the rod as observed in frame S′, and briefly justify your answer. Indicate any assumptions which you have made.

(v) By assuming that the force on a charge due to electric and magnetic fields is *form invariant* between inertial frames, show that the force, $\mathbf{F}'$, exerted on the test charge by the fields in S′ is given by

$$\mathbf{F}' = \mathbf{F}/\sqrt{1 - V^2/c^2}.$$

(vi) *If we accept the principle of relativity, Newton's second law has to be sacrificed and his ideas of space and time re-examined.*

Explain what is meant by this statement using no more than 250 words. Refer directly to previous parts of this question to illustrate points in your answer where appropriate.

Question 2

This question carries 20% of the marks for this assignment.

(i) Explain what is meant by time dilation in special relativity, giving an unambiguous description of the way in which different observers disagree about the time interval between successive ticks on a given clock.

(ii) Deduce the time dilation formula by exploiting the invariance of the Lorentz invariant interval.

(iii) *Moving clocks run slow, but an observer may see a moving clock as running fast or slow.* Do you agree with this statement? If you do, give reasons for your opinion and indicate the precise conditions under which a moving clock will be seen to run fast.

Question 3

This question carries 35% of the marks for this assignment.

(i) A particle of mass m, momentum $\mathbf{p}$ and energy E decays entirely into two pulses of radiation of energies E_1 and E_2 which emerge in directions making an angle θ with each other. Write down expressions for E and $|\mathbf{p}|^2$ in terms of E_1, E_2 and θ and show that

$$2E_1E_2(1 - \cos\theta) = m^2c^4.$$