The Open University Supplementary Material
S357 Space, Time and Cosmology

Course and assignment number:

Tutor Marked Assignment

S357 01

Make sure you know how to complete and send in your TMA and PT3 form: detailed instructions are given in your student handbook (or supplement).

Covering: **Units 1–3**

Cut-off date:
Friday 4 April 1997

This assignment consists of a single question with nine parts (i) (ix).

Question 1

This question carries 100% of the marks for this assignment.

(i) Describe, in no more than 250 words, what is meant by an inertial frame of reference, explaining why the concept is important in Newtonian physics. Your answer should include an account of a method by which an inertial frame can be defined operationally.

(28 marks)

(ii) Two particles with masses m_1 and m_2 at positions $\mathbf{x}_1$ and $\mathbf{x}_2$, form an isolated system, i.e. they exert forces on each other but have negligible interactions with the rest of their surroundings. Use Newton's laws of motion to show that the centre of mass $\mathbf{X}$ of the particles, defined by

$$\mathbf{X} = \frac{m_1\mathbf{x}_1 + m_2\mathbf{x}_2}{m_1 + m_2},$$

undergoes unaccelerated motion in any *inertial* reference frame.

(10 marks)

Parts (iii)–(ix) concern an isolated system comprising two small bodies, A and B, each of mass 0.5 kg. Show your working in all the calculations.

(iii) The motions of A and B are observed in a non-rotating reference frame S′, and their positions at time t (in seconds) are given in metres by

$$\mathbf{x}'_{\mathrm{A}}(t) = (2\cos 2t, \tfrac{1}{2}\sin 2t, t^2)$$

and

$$\mathbf{x}'_{\mathrm{B}}(t) = (0, -\tfrac{1}{2}\sin 2t, t^2).$$

Using the results of part (ii), or otherwise, show that S′ must be non-inertial. Explain your reasoning carefully.

(6 marks)

(iv) A second reference frame, S, has its axes parallel to those of S′, and its origin permanently located at the centre of mass of the two bodies. Is S inertial? Briefly justify your answer.

(4 marks)

(v) Calculate the position vector, $\mathbf{x}_{\mathrm{A}}(t)$, and the velocity vector, $\mathbf{v}_{\mathrm{A}}(t)$ of body A referred to the frame S. Do the same for body B.

Write down an expression for the total momentum $\mathbf{P}$ of the system in S, in terms of the velocity vectors $\mathbf{v}_{\mathrm{A}}$ and $\mathbf{v}_{\mathrm{B}}$, and calculate its value.

(10 marks)

(vi) Write down an expression for the total angular momentum $\mathbf{J}$ of the system in S in terms of $\mathbf{x}_{\mathrm{A}}$, $\mathbf{x}_{\mathrm{B}}$, $\mathbf{v}_{\mathrm{A}}$ and $\mathbf{v}_{\mathrm{B}}$, and calculate its value.

(6 marks)

(vii) Show that the forces acting on each of A and B in S are proportional to the displacement vector between them.

Explain, in very general terms, why you would not expect these forces to be able to account for the observed motions of the bodies in S′.

(12 marks)

(viii) Given that the potential energy function of the system in S is

$$U = \tfrac{1}{2}|\mathbf{x}_{\mathrm{A}} - \mathbf{x}_{\mathrm{B}}|^2$$

(in joules), obtain an expression for the total energy function, E, of the system in terms of $\mathbf{x}_{\mathrm{A}}$, $\mathbf{x}_{\mathrm{B}}$, $\mathbf{v}_{\mathrm{A}}$ and $\mathbf{v}_{\mathrm{B}}$. Calculate the value of E.

(10 marks)

(ix) Let $\mathbf{P}'$, $\mathbf{J}'$ and E' be given by the *same* expressions involving the position and velocity vectors of A and B as $\mathbf{P}$, $\mathbf{J}$ and E, in parts (v), (vi) and (viii), but using position and velocity vectors $\mathbf{x}'_{\mathrm{A}}$, $\mathbf{x}'_{\mathrm{B}}$, $\mathbf{v}'_{\mathrm{A}}$ and $\mathbf{v}'_{\mathrm{B}}$, referred to S′ rather than S. Calculate the values of $\mathbf{P}'$, $\mathbf{J}'$ and E', and comment on how they differ from the values of $\mathbf{P}$, $\mathbf{J}$ and E obtained earlier.

(14 marks)

$2\frac{1}{2}\cos^2 2t + 4\frac{1}{2}\sin^2 2t$