S357 Space, Time and Cosmology
S357, CMA 42 Answers (1997)

PART A

Q1 [E]

$$(c\Delta t')^2 - |\Delta x'|^2 = (c\Delta t)^2 - |\Delta x|^2$$
$$(2.6 \times 3 \times 10^5\,\text{km})^2 - |\Delta x'|^2 = (2.5 \times 3 \times 10^5\,\text{km})^2 - (2.1 \times 10^5\,\text{km})^2$$

So, $|\Delta x'| = 3 \times 10^5$ km.

Q2 [B,C] In the observer's units

$$(S_{ab})^2 = 2^2 - 1^2 = 3$$
$$(S_{ac})^2 = 1^2 - 4^2 = -15$$
$$(S_{bc})^2 = 3^2 - 3^2 = 0$$

In any other inertial frame, using the same units

$$(c\Delta t'_{ab})^2 - (\Delta x'_{ab})^2 = 3, \text{so} \quad |\Delta t'_{ab}| \geqslant \frac{\sqrt{3}}{c}$$
$$(\Delta x'_{ac})^2 - (c\Delta t'_{ac})^2 = 15, \quad \text{so} \quad |\Delta x'_{ac}| \geqslant \sqrt{15}$$
$$c|\Delta t'_{bc}| = |\Delta x'_{bc}|$$

A is false since $|\Delta t'_{ab}| \neq 0$ in any inertial frame.
B is true as $|\Delta x'_{ab}|$ can be zero.
C is true as $|\Delta t'_{ac}|$ can be zero.
D is false as $|\Delta x'_{ac}| \neq 0$.
E & F would require an inertial observer to travel at speed c and are therefore false.

Q3 [B,D,E]

Using the observer's units A is true.
B is false since $\mathscr{E}_c$ could not cause $\mathscr{E}_a$.
C is true; $|\Delta t'_{ab}| \geqslant \frac{\sqrt{3}}{c}$ and $1/c < \sqrt{3}/c$.
D is false; $2/c > \sqrt{3}/c$.
E is false; $|\Delta x'_{ac}| \geqslant \sqrt{15}$ and $4 > \sqrt{15}$.
F is true; $3 < \sqrt{15}$.

Q4 [C,F] The invariant interval between the two explosions is

$$s^2 = c^2\Delta t^2 - |\Delta x|^2 = (9 \times 10^{16} \times 100 - 4 \times 10^{18})\,\text{m}^2$$
$$= 5 \times 10^{18}\text{m}^2 \text{(to one significant figure)}.$$

The event pairs recorded by A, B and D agreee with this interval and may, therefore, describe the same events. C disagrees and is therefore wrong. E and F have recorded time more accurately, but the invariant interval found by E is still $5 \times 10^{18}\text{m}^2$ to one significant figure and therefore agrees with that of the original explosions whereas that of F does not.