

Q5 [C] Using Equation 25 of Unit 7, and SAQ 20

$$\frac{f_0}{f} = \gamma \left(1 - \frac{u}{c} \cos \theta\right)$$

where $\gamma = 1/\sqrt{1 - \frac{u^2}{c^2}}$

$$u = 0.153c, \gamma = 1.012 \quad 1 - \frac{u}{c} \cos \theta = 1 - 0.141 = 0.859$$

$$\text{So, } f_0 = 5.88 \times 10^{14} \times 1.012 \times 0.859 \text{ Hz} = 5.11 \times 10^{14} \text{ Hz.}$$

Q6 [A] Let mass of ion be m , and speed v . Then

$$E = \gamma mc^2 \text{ and } p = \gamma mv.$$

$$\begin{aligned} E^2 - p^2 c^2 &= \gamma^2 m^2 c^4 - \gamma^2 m^2 v^2 c^2 = m^2 \gamma^2 c^4 (1 - v^2/c^2) \\ &= m^2 c^4 = (12.5)^2 u^2 c^4 - (3.5)^2 u^2 c^4 \\ &= 144 u^2 c^4 \end{aligned}$$

So,

$$m = \sqrt{144} u = 12u.$$

Q7 [E]

$$\frac{pc}{E} = \frac{\gamma mcv}{\gamma mc^2} = \frac{v}{c} = \frac{3.5 uc^2}{12.5 uc^2} = 0.28.$$

Q8 [C] Let E_γ be the energy of the radiation, then the energy of the emitted particle is $15mc^2 - E_\gamma$. From the hint, the momentum magnitude of the radiation is

$$\frac{E_\gamma}{c}$$

and this is also the momentum magnitude of the particle. Thus, applying the relation $E^2 = p^2 c^2 + m^2 c^4$ to the emitted particle,

$$(14mc^2)^2 + E_\gamma^2 = (15mc^2 - E_\gamma)^2$$

So,

$$E_\gamma = \frac{29}{30} mc^2.$$

Q9 [C] Using the result of Q8 and the general expression for the momentum magnitude of a particle of known mass and speed;

$$14 m \gamma v = \frac{E_\gamma}{c} = \frac{29}{30} mc$$

so

$$\frac{\gamma v}{c} = \frac{29}{30 \times 14} = \frac{v/c}{\sqrt{1 - (v/c)^2}}$$

i.e.

$$\frac{v}{c} = 6.888 \times 10^{-2}, v = 2.067 \times 10^7 \text{ ms}^{-1}.$$

Q10 [C] The proper time between two events does, in general, depend on the world-line along which it is measured. Thus statement C is wrong. All the other statements are supported by Section 5 of Unit 7, apart from D which requires you to know that different events on the world-line of a material particle cannot have zero invariant separation.