Tutor Marked Assignment

Course and assignment number:
S271 04

Make sure you know how to complete and send in your TMA and PT3 form: detailed instructions are given in your student handbook (or supplement).

Covering: **Units 13–15**

Cut-off date:
Friday 20 September 1996

This assignment consists of three questions.

Question 1

This question is mainly related to Unit 13 and carries 38% of the marks for this assignment.

Bohr's quantum model successfully accounts for the size and the spectroscopy of the hydrogen atom, which consists of an electron moving around a proton. There are many similarities between the electrostatic forces that bind an electron to a proton and the gravitational forces that maintain the planets in orbit around the Sun, and a satellite in orbit around the Earth. Both forces obey an attractive *inverse-square law*. It is therefore possible to adapt Bohr's postulates and his calculations to obtain a gravitational analogue of the Bohr model of the hydrogen atom, and this is what you will do in answering this question. In particular, you are asked to set up a simple quantum model to describe a satellite, mass m_s, that travels at speed v in a (circular) orbit, radius R, around the Moon, mass M. You should ignore gravitational forces between the satellite and all other bodies apart from the Moon.

(a) Write down adapted forms of Bohr's four postulates that apply to the satellite orbiting the Moon. (*5 marks*)

(b) Show that the allowed radii R_n of the orbits of the satellite are given by $R_n = n^2h^2/4\pi^2GMm_s^2$, where R_n is greater than the Moon's radius and n is a (suitably large) positive integer. (*11 marks*)

(c) Show that the speed v_n of the satellite in the orbit with radius R_n is given by $v_n = 2\pi GMm_s/nh$. (*7 marks*)

(d) Now assume that the satellite has mass 5.0×10^4 kg and is in a circular orbit 200 km above the Moon's equatorial surface. The Moon's radius is 1 720 km and its mass is 7.2×10^{22} kg. Calculate the approximate value of the quantum number n for this orbit, and hence calculate the value of the speed of the satellite. (*5 marks*)

(e) Calculate $(v_n - v_{n+1})/v_n$, the fractional change in speed when the satellite makes a transition from an orbit with quantum number n (as calculated in part d) to an orbit with quantum number $n + 1$. State the correspondence principle (Unit 14), and discuss (in a few sentences) how this principle applies to your result for $(v_n - v_{n+1})/v_n$ obtained with the 'Bohr satellite model'. (*10 marks*)

Hint: You can expect to obtain a very large number as your answer for part (d). This may restrict your use of a calculator in part (e): if so, use the fact that $\frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)}$.

Question 2

This question is related to Unit 14, and carries 25% of the marks for this assignment.

You saw in Unit 6 how Newtonian mechanics is used to describe the motion of a particle of mass M that is attached to a spring of force constant k_s and undergoes simple harmonic motion along the x-axis about an equilibrium position, $x = 0$, with amplitude x_{max}. This question concerns the Schrödinger equation and the way it is used to give a *quantum mechanical* description of the same system.

(a) Write down Schrödinger's equation for this system when its total energy is E_n. Your answer should be an equation relating to the wavefunction ψ and $d^2\psi/dx^2$ to the position coordinate x and the constants M, k_s, $\hbar$ and E_n. [*Health warning*: do not try to solve this equation!] (*6 marks*)

(b) Suppose that a certain wavefunction ψ_n is a satisfactory solution of the Schrödinger equation you wrote down in part (a). State in a few sentences what can then be said about the wavefunction ψ_n and the value E_n. (*6 marks*)

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