

other words they are parameters of the model. Since only the separation of the protons can be physically significant, the single adjustable parameter of the model is in fact  $|r_1 - r_2| = R$ . The proton-proton electrostatic energy  $E_{pp} = (e^2/4\pi\epsilon_0)/R$  does not depend on  $r$  and can be subtracted from the reduced one-body Schrodinger equation.

The resulting hamiltonian is shown in box 3.

The BOpp TISE is

$$\mathbf{H}_e \psi_e = E_e \psi_e$$

$\psi_e(r)$  is the electron wave function and  $E_e$  is the energy of the electron. The BOpp approximation to the total energy is then  $E_{\text{tot}} = E_e + E_{pp}$ .

The Born Oppenheimer model now describes an electron moving in a double well potential,

$$V_e = -(e^2/4\pi\epsilon_0)(|r_1 - r|^{-1} + |r_2 - r|^{-1})$$

$V_e$  is not a central potential. This makes the model hard to solve, and we move on to the approximation scheme of box 4.

The **Variational solution for  $E_e(R)$**  follows another sequence which is contained in box 4.

A sensible choice for  $\psi_t$  is important in variational methods, and in this case a superposition of 1s hydrogen wave functions is a good place to start. This is because when the electron is near to either proton, the system imitates a hydrogen atom and a free proton. Using  $\psi_1$  and  $\psi_2$  to denote 1s hydrogen wave functions centred on either proton then the variational choice is:

$$\psi_t = A\psi_1 + B\psi_2.$$

$E_{e,t}(A,B)$  is minimised at an extremum:  $(\partial/\partial A) E_{e,t}(A,B) = (\partial/\partial B) E_{e,t}(A,B) = 0$ . There are two solutions,  $A = B$  or  $A = -B$ , a result which also follows from the symmetry of the system ( $|A|^2 = |B|^2$ ). The two solutions have even (+) and odd parity (-):

$$\psi_+ = N_+ (\psi_1 + \psi_2)$$

and

$$\psi_- = N_- (\psi_1 - \psi_2)$$

$N_{+/-}$  normalise the solutions. The energies calculated at these extrema are  $E_{+/-}$ . The process is looped for different values of the parameter  $R$ .