THE HYDROGEN ATOM

INTRODUCTION

The simplest model for the hydrogen atom is that of a central proton beyond which is an electron moving under the action of a Coulombic potential.

The analysis of the hydrogen atom is then concerned with a central force in which the potential energy $V(r)$ is a function solely of distance and may be written: $V(r) = \frac{-e^2}{4\pi\varepsilon_0} \cdot \frac{1}{r}$

With such a simple model we are making assumptions such as:

(i) The nucleus moves so slowly relative to the electron that it has negligible effect on the behaviour of the electron (the Born-Oppenheimer Principle) and may be considered to be fixed at the origin.

(ii) Relativistic effects are so small (0.001% of the total energy of the system) that they need not be considered except when discussing fine structure in spectra.

Similarly we need not consider other small influences such as the spin-orbit interaction (and Lamb shift) and the Zeeman effect.

SCHRÖDINGER EQUATION

With the assumptions made the Schrödinger equation may be written:

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi + V(r)\psi = E\psi \qquad (1)$$

The spherical symmetry of the potential energy suggests the use of spherical polar coordinates (r, θ, ϕ) such that $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$.

Equation (1) then takes the form: