2) $(P, \underline{a}, \hbar/2) = |\underline{e}_-^\dagger \underline{a}|^2$

$$\underline{e}_- = \begin{pmatrix} -\sin(\pi/4)\, e^{-\pi i/2} \\ \cos(\pi/4) \end{pmatrix} = \begin{pmatrix} i/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$P = \left| \left(-i/\sqrt{2} \quad 1/\sqrt{2} \right) \begin{pmatrix} \sqrt{2}/\sqrt{5} \\ \sqrt{3}/\sqrt{5} \end{pmatrix} \right|^2$$

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$$= \left| \frac{-i}{\sqrt{5}} + \frac{\sqrt{3}}{\sqrt{10}} \right|^2 = \left(\frac{1}{5} + \frac{3}{10} \right) = \frac{1}{2}$$

3) i) a) $\Delta(A)_{\underline{\psi}} \Delta(B)_{\underline{\psi}} \geq \frac{1}{2} |(\underline{\psi}, (\hat{A}\hat{B} - \hat{B}\hat{A})\underline{\psi})|$

see back of this page.

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~~The Equation above relates the precision to which two observable physical properties can simultaneously be measured to the commutator $\hat{A}\hat{B} - \hat{B}\hat{A}$. If $\hat{A}, \hat{B}$ do not commute, so that $\hat{A}\hat{B} - \hat{B}\hat{A} \neq 0$, then in general A and B can not be measured to an arbitrarily high precision simultaneously, so there exists an uncertainty relation between eg x, P.~~

Put $\hat{A} = \hat{x}$, $\hat{B} = \hat{p}$

$$\Delta(x)_{\underline{\psi}} \Delta(p)_{\underline{\psi}} \geq \frac{1}{2} |(\underline{\psi}(\hat{x}\hat{p} - \hat{p}\hat{x})\underline{\psi})|$$

$$= \frac{1}{2} \left| \left(\underline{\psi} \left(x \left(-i\hbar \frac{\partial}{\partial x} \right) - \left(-i\hbar \frac{\partial}{\partial x} \right) x \right) \underline{\psi} \right) \right|$$

$$= \frac{1}{2} \left| \left(\underline{\psi} \left(-i\hbar x \frac{\partial}{\partial x} + i\hbar \quad ?? \quad i\hbar x \frac{\partial}{\partial x} \right) \underline{\psi} \right) \right|$$

$$= \frac{\hbar}{2} |(\underline{\psi}, \underline{\psi})|$$

$$= \frac{\hbar}{2} \left| \int_{-\infty}^{\infty} \underline{\psi}^* \underline{\psi} \, dx \right|$$

$$= \frac{\hbar}{2}$$

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ie $\Delta(x)_{\underline{\psi}} \Delta(p)_{\underline{\psi}} \geq \frac{\hbar}{2}$

ii) Suppose a wavefunction $\underline{\psi}$ is in an eigenstate of an operator $\hat{A}$ representing an observable A, with eigenvalue λ. Then $\Delta A = (\langle A^2 \rangle - \langle A \rangle^2)^{1/2}$

For simplicity assume one dimension

$$\langle A^2 \rangle_{\underline{\psi}} = \int_{-\infty}^{\infty} \underline{\psi}^* \hat{A}^2 \underline{\psi} \, dx$$

$$= \int_{-\infty}^{\infty} \underline{\psi}^* \hat{A} A \underline{\psi} \, dx$$

none is this is necessary! For an eigenstate of $\hat{A}$ the $\Delta(A) = 0$