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to look for quantization of angular momentum in neutral atoms.

b) True $\mu_e = \frac{e\hbar}{2m_e}$ $\mu_n = \frac{e\hbar}{2m_n}$

$m_n > m_e$ by a factor of ~ 1840

$\therefore \mu_e > \mu_n$ by a factor of about 1840

not required if they are true.

c) Yes. Suppose ψ is an eigenstate of $\hat{L}_z$ with eigenvalue zero, then if ψ is also an eigenstate of $\hat{L}_x$ with eigenvalue zero the operators will commute, since

$(\hat{L}_z\hat{L}_x - \hat{L}_x\hat{L}_z)\psi = \hat{L}_z\hat{L}_x\psi - \hat{L}_x\hat{L}_z\psi$
$= \hat{L}_z 0 - \hat{L}_x 0 = 0 - 0 = 0$

But since $(\hat{L}_z\hat{L}_x - \hat{L}_x\hat{L}_z) = i\hbar\hat{L}_y$, the state is also an eigenstate of $\hat{L}_y$ with eigenvalue zero. This is the only possibility.

ditto

d) No. For this to be possible $\hat{L}_x$ and $\hat{L}_z$ would have to commute ie $(\hat{L}_z\hat{L}_x - \hat{L}_x\hat{L}_z)\psi = 0$

but $(\hat{L}_z\hat{L}_x - \hat{L}_x\hat{L}_z) = i\hbar\hat{L}_y$

so $\hat{L}_x$ and $\hat{L}_z$ do not commute and simultaneous eigenstates are impossible. (From c) the only eigenstate simultaneously eigenstates of $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ is that with eigenvalue zero for all three operators).

e) Yes. The potential is of the form $\frac{Cr^2}{2}$ and some possible eigenvectors are given in FT page 462

ditto

eg $\psi = k\, e^{i\phi}\, r\, e^{-r^2/2a^2}$

is an eigenfunction of $\hat{L}_z$ and $\hat{E}$ (see opposite)

f) Yes $(\hat{L}_z\hat{L}_z^2 - \hat{L}_z^2\hat{L}_z)\psi$

ditto

$= (\hat{L}_z\hat{L}_z\hat{L}_z - \hat{L}_z\hat{L}_z\hat{L}_z)\psi = 0$

$\hat{L}_z$ and $\hat{L}_z^2$ commute $\therefore$ they may have simultaneous eigenfunctions