③

so $E_0 = (0+\frac{1}{2})\hbar\omega = \frac{\hbar\omega}{2}$

$E_2 = (2+\frac{1}{2})\hbar\omega = \frac{5\hbar\omega}{2}$

$\therefore \langle E \rangle = \frac{1}{5} \times \frac{\hbar\omega}{2} + \frac{4}{5} \times \frac{5\hbar\omega}{2}$ ✓

$= \frac{21\hbar\omega}{10}$

5/5

d) We must first find $\langle E^2 \rangle$. By a process almost identical to that in c) we can derive the equation,

$\langle E^2 \rangle = \sum_{n=0}^{\infty} |a_n|^2 E_n^2$ ✓

since the E_n are eigenvalues (so $H\psi_n = E_n\psi_n$, and $H^2\psi_n = E_n^2\psi_n$)

so $\langle E^2 \rangle = |a_0|^2 E_0^2 + |a_2|^2 E_2^2$

$= \left|\frac{1}{\sqrt{5}}\right|^2 \times \left(\frac{\hbar\omega}{2}\right)^2 + \left|\frac{2i}{\sqrt{5}}\right|^2 \times \left(\frac{5\hbar\omega}{2}\right)^2$

$= \frac{\hbar^2\omega^2}{20} + \frac{4}{5} \times \frac{25\hbar^2\omega^2}{4}$

$= \frac{101\hbar^2\omega^2}{20}$ ✓

then $\Delta E = (\langle E^2 \rangle - \langle E \rangle^2)^{\frac{1}{2}}$

$= \left(\frac{101\hbar^2\omega^2}{20} - \left(\frac{21\hbar\omega}{10}\right)^2\right)^{\frac{1}{2}}$ ✓

$= \left(\left(\frac{101}{20} - \frac{441}{100}\right)\hbar^2\omega^2\right)^{\frac{1}{2}}$

$= \left(\frac{505 - 441}{100}\right)^{\frac{1}{2}} \hbar\omega$ ✓

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$= \sqrt{\frac{64}{100}}\hbar\omega = \frac{4\hbar\omega}{5}$