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c) $\Psi(x,t) = \frac{\psi_0(x)}{\sqrt{5}}\exp\left(\frac{-i\omega t}{2}\right) + \frac{2i}{\sqrt{5}}\psi_2(x)\exp\left(\frac{-5i\omega t}{2}\right)$

The probability density is given by $|\Psi(x,t)|^2 = \Psi^*(x,t)\,\Psi(x,t)$ ✓

the ψ_n are real for the simple harmonic oscillator, so $\psi_n^* = \psi_n$.

$$\left(\frac{\psi_0(x)}{\sqrt{5}}e^{+\frac{i\omega t}{2}} - \frac{2i}{\sqrt{5}}\psi_2(x)e^{\frac{5i\omega t}{2}}\right)\left(\frac{\psi_0(x)}{\sqrt{5}}e^{-\frac{i\omega t}{2}} + \frac{2i\psi_2(x)}{\sqrt{5}}e^{-\frac{5i\omega t}{2}}\right)$$

$$= \frac{|\psi_0(x)|^2}{5} + 4\frac{|\psi_2(x)|^2}{5}$$
$$+\left(\frac{2i}{5}e^{-2i\omega t} - \frac{2i}{5}e^{2i\omega t}\right)\psi_0(x)\psi_2(x) \quad ✓$$

$$= \frac{|\psi_0(x)|^2}{5} + 4\frac{|\psi_2(x)|^2}{5}$$
$$+ \frac{4}{5}\left(\frac{e^{2i\omega t} - e^{-2i\omega t}}{2i}\right)\psi_0(x)\psi_2(x) \quad ✓$$

$$= \frac{|\psi_0(x)|^2}{5} + 4\frac{|\psi_2(x)|^2}{5} + \frac{4}{5}\psi_0(x)\psi_2(x)\sin 2\omega t$$

5/5 ∴ the probability density oscillates sinusoidally with time, with angular frequency 2ω. ✓

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