

4(a) Both  $e^{2i\phi}$  and  $e^{-2i\phi}$  are eigenfunctions of  $\hat{L}_z = \left(-i\hbar \frac{\partial}{\partial \phi}\right)$

with eigenvalues  $2\hbar$  and  $-2\hbar$  respectively.

Measurements of  $L_z$  on this system will yield these two values with equal probability (because their coefficients are the same)

$$\text{Hence } \langle L_z \rangle = \frac{1}{2} [2\hbar + (-2\hbar)] = 0$$

$$\text{Similarly } \langle L_z^2 \rangle = \frac{1}{2} [(2\hbar)^2 + (-2\hbar)^2] = 4\hbar^2$$