SM355 TMA 02

1)a) Since $|\Psi(x,t)|^2 \Delta x$ is the probability (approx) of finding the particle between x and $x+\Delta x$, and since the particle must be somewhere, by letting Δx tend to zero we can say

$$dP = |\Psi(x,t)|^2 dx$$

then

$$1 = \int dP = \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx$$

$$= \int_{-\infty}^{\infty} \Psi^*(x,t)\Psi(x,t)\,dx \quad \text{for all } t.$$

At $t=0$, for Ψ given

$$1 = \int_{-\infty}^{\infty} N^* N (\psi_0^*(x) - 2i\psi_2^*(x))(\psi_0(x) + 2i\psi_2(x))\,dx$$

$$= \int_{-\infty}^{\infty} |N|^2 (\psi_0^*(x)\psi_0(x) + 4\psi_2^*(x)\psi_2(x) + 2i(\psi_0^*(x)\psi_2(x) - \psi_2^*(x)\psi_0(x)))\,dx$$

eigenfunctions ψ_n are orthonormal

ie $\int \psi_n^* \psi_m \, dx = \begin{cases} 0 & n \neq m \\ 1 & n = m \end{cases}$

So the last bracketed term in the above integral is zero

$$1 = |N|^2 \int_{-\infty}^{\infty} \psi_0^*(x)\psi_0(x) + 4\psi_2^*(x)\psi_2(x)\,dx$$

$$= |N|^2(1+4) = 5|N|^2$$

$$1 = 5|N|^2 \Rightarrow |N| = \frac{1}{\sqrt{5}} \Rightarrow N = \frac{e^{i\delta}}{\sqrt{5}}$$

5/5 Take $\delta = 0$, since it is arbitrary, then $N = \frac{1}{\sqrt{5}}$.

b) The time dependence of the harmonic oscillator soln for ψ_n is $\exp(-iE_n t/\hbar)$ where E_n is the energy eigenvalue for ψ_n, $E_n = (n+1/2)\hbar\omega$

$$\therefore \Psi(x,t) = \sum a_n \psi_n(x) \exp(-iE_n t/\hbar)$$

but $a_n = 0$ if $n \neq 0, 2$.

$$\therefore \Psi(x,t) = a_0 \psi_0(x) \exp(-iE_0 t/\hbar) + a_2 \psi_2(x) \exp(-iE_2 t/\hbar)$$