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$$= \frac{+4\hbar^2}{\pi}\int_0^{\infty}\int_0^{2\pi} R^2(r)\, r \,\frac{1}{2}(1+\cos 4\phi)\, d\phi\, dr$$

$$= \frac{+4\hbar^2}{\pi}\int_0^{\infty}\left[R^2(r)\, r\,\frac{1}{2}\left(\phi + \frac{1}{4}\sin 4\phi\right)\right]_0^{2\pi} d\phi\, dr$$

$$= \frac{4\hbar^2}{\pi}\int_0^{\infty} R^2(r)\, r \times \frac{1}{2}\left(2\pi + \frac{\sin 8\pi}{4} - 0 - \frac{\sin 0}{4}\right) dr$$

$$= \frac{4\hbar^2}{\pi}\int_0^{\infty} R^2(r) \times \pi \, dr$$

$$= 4\hbar^2 \int_0^{\infty} R^2(r)\, r\, dr = 4\hbar^2$$

using the integral on the question paper.

$$\Delta(L_z) = (\langle L_z^2\rangle - \langle L_z\rangle^2)^{1/2}$$

$$= (4\hbar^2 - 0^2)^{1/2} = 2\hbar$$

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