2

$a_0 = \frac{1}{\sqrt{5}}$, $a_2 = \frac{2i}{\sqrt{5}}$ from a)

5/5 so $\Psi(x,t) = \frac{\psi_0(x)}{\sqrt{5}} \exp\left(-\frac{i\omega t}{2}\right) + \frac{2i}{\sqrt{5}} \psi_2(x) \exp\left(-\frac{5i\omega t}{2}\right)$

c) $\langle E \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\, \hat{H}\, \Psi(x,t)\, dx$

this is unnecessarily complicated when there are only 2 terms

$$= \int_{-\infty}^{\infty} \left(\sum_{n=0}^{\infty} a_n^* \psi_n^*(x) e^{+\frac{iE_n t}{\hbar}}\right) \hat{H} \left(\sum_{m=0}^{\infty} a_m \psi_m(x) e^{-\frac{iE_m t}{\hbar}}\right) dx$$

$$= \int_{-\infty}^{\infty} \left(\sum_{n=0}^{\infty} a_n^* \psi_n^*(x) e^{+\frac{iE_n t}{\hbar}}\right) \left(\sum_{m=0}^{\infty} a_m \hat{H} \psi_m(x) e^{-\frac{iE_m t}{\hbar}}\right) dx$$

$$= \int_{-\infty}^{\infty} \left(\sum_{n=0}^{\infty} a_n^* \psi_n^*(x) e^{+\frac{iE_n t}{\hbar}}\right) \left(\sum_{m=0}^{\infty} a_m E_m \psi_m(x) e^{-\frac{iE_m t}{\hbar}}\right) dx$$

(since the ψ_m are the eigenfunctions of the simple harmonic oscillator.)

We can write the above equation

$$\langle E \rangle = \int_{-\infty}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_n^* a_m E_m \psi_n^*(x) \psi_m(x) e^{\frac{i(E_n - E_m)t}{\hbar}}\, dx$$

$$\langle E \rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_n^* a_m E_m e^{\frac{i(E_n - E_m)t}{\hbar}} \int_{-\infty}^{\infty} \psi_n^*(x) \psi_m(x)\, dx$$

By orthonormality of eigenfunctions spatial parts

$$\int_{-\infty}^{\infty} \psi_n^*(x) \psi_m(x)\, dx = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}$$

You could start from here & get full marks

$$\therefore \langle E \rangle = \sum_{n=0}^{\infty} a_n^* a_n E_n = \sum_{n=0}^{\infty} |a_n|^2 E_n$$

$a_0 = \frac{1}{\sqrt{5}}$, $a_2 = \frac{2i}{\sqrt{5}}$, $a_n = 0$ if $n \neq 0, 2$

$$\therefore \langle E \rangle = \left|\left(\frac{1}{\sqrt{5}}\right)\right|^2 \times E_0 + \left|\frac{2i}{\sqrt{5}}\right|^2 E_2$$

the E_n are the harmonic oscillator energy eigenvalues $E_n = (n + 1/2)\hbar\omega$.