ie the turning points of the motion are at $x = \pm\sqrt{3}\,a$ so that $-a\sqrt{3} \le x \le a\sqrt{3}$.

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c) The given solution for ψ is the 1st excited state. The probability distribution is the graph of the function $|\psi(x)|^2$ ie $N^2\left(\frac{4x^2}{a^2}\right)\exp\left(\frac{-x^2}{a^2}\right)$

This is symmetric about 0 and tends to 0 as $x \to \pm\infty$. Also, $|\psi(0)|^2 = 0$.

$$\frac{d}{dx}\left(\frac{x^2}{a^2}\exp\left(\frac{-x^2}{a^2}\right)\right) = \left(\frac{2x}{a^2} - \frac{2x^3}{a^4}\right)\exp\left(\frac{-x^2}{a^2}\right)$$

which equals 0 when $\frac{2x}{a^2} - \frac{2x^3}{a^4} = 0$

ie $x = a$ or $x = 0$ and as $x \to \pm\infty$.

not required

when $x = \pm a\sqrt{3}$,

$$|\psi(x)|^2 = N^2\left(\frac{4\times(\pm a\sqrt{3})^2}{a^2}\right)\exp\left(\frac{-(\pm a\sqrt{3})^2}{a^2}\right)$$

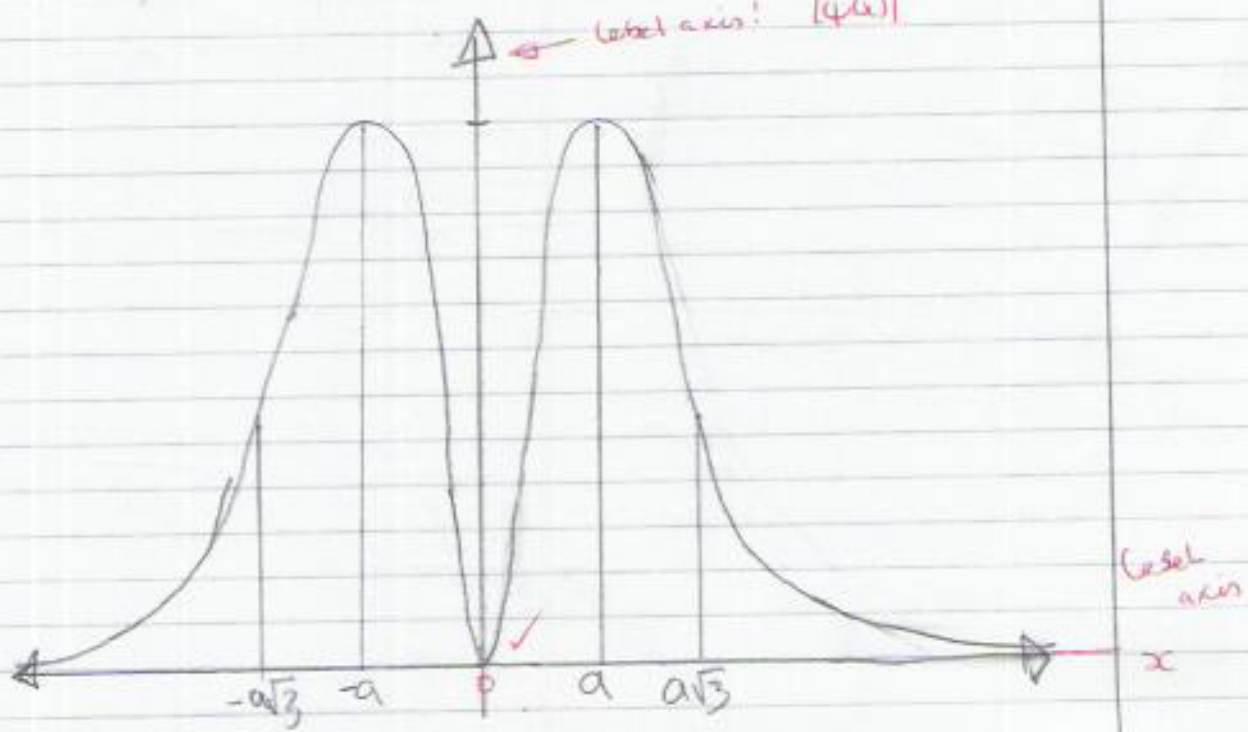
$$= N^2 \times 12\exp(-3)$$

when $x = \pm a$

$$|\psi(x)|^2 = N^2\left(\frac{4\times a^2}{a^2}\right)\exp\left(\frac{-a^2}{a^2}\right)$$

$$= N^2 \times 4\exp(-1)$$

$$\frac{|\psi(\pm a\sqrt{3})|^2}{|\psi(\pm a)|^2} = \frac{N^2\times 12\exp(-3)}{N^2\times 4\exp(-1)} = 3\exp(-2)$$

$$= 0.406$$

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