$$\langle V^2\rangle_0 = 4\pi\int_0^\infty r^2\, u_0(r)\, V^2(r)\, dr$$

Now $|u_0(r)|^2 = \dfrac{r^2}{\pi a_0^3}\exp(-2r/a_0)$ (Unit 4)

& $E_1^2 = \left(\dfrac{e^2}{8\pi\varepsilon_0 a_0}\right)^2$ & $V(r) = -\dfrac{e^2}{4\pi\varepsilon_0 r}$.

$\therefore$ ~~$\langle V^2\rangle_0 = 4\pi\int_0^\infty \frac{e^4}{16\pi^2\varepsilon_0^2 r^2}\frac{r^2}{\pi a_0^3}$~~ Sorry!

$$\therefore \langle V^2\rangle_0 = 4\pi\int_0^\infty \frac{e^4}{16\pi^2\varepsilon_0^2 r^2}\cdot\frac{r^2}{\pi a_0^3}\exp\left(-2r/a_0\right)dr.$$

$$\therefore \frac{\langle V^2\rangle_0}{E_1^2} = \frac{e^4}{4\pi^2\varepsilon_0^2 a_0^3}\cdot\frac{64\pi^2\varepsilon_0^2 a_0^2}{e^4}\int_0^\infty \exp\left(-\frac{2r}{a_0}\right)dr$$

$$= \frac{16}{a_0}\int_0^\infty \exp\left(-\frac{2r}{a_0}\right)dr$$

Then put $\rho = r/a_0$ So $dr = a_0\, d\rho$

Then $\dfrac{\langle V^2\rangle_0}{E_1^2} = 16\displaystyle\int_0^\infty \exp(-2\rho)\, d\rho = -8\left[\exp(-2\rho)\right]_0^\infty = 8$

ie. $\langle V^2\rangle_0 = 8E_1^2$