In this case $\alpha T'' + (\beta - \lambda) T = 0$

$\lambda = -k^2 = -\left(\frac{\pi n}{L}\right)^2$

$\alpha T'' + \left(\beta + \left(\frac{\pi n}{L}\right)^2\right) T = 0$

$T'' + \frac{1}{\alpha}\left(\beta + \frac{\pi^2 n^2}{L^2}\right) T = 0$

$\alpha, \beta > 0$ implies a sinusoidal soln

$T = C_n \cos \omega_n t + D_n \sin \omega_n t$

with $\omega_n = \sqrt{\frac{1}{\alpha}\left(\beta + \frac{\pi^2 n^2}{L^2}\right)}$ ✓

Then $Y_n = X_n(x) T_n(t)$

$= B_n \sin k_n x \,(C_n \cos \omega_n t + D_n \sin \omega_n t)$

$= \sin k_n x \,(A_n \cos \omega_n t + B_n \sin \omega_n t)$

not a good choice of constant!

where $A_n = B_n C_n$, $B_n = B_n D_n$

18/18

But Y_n is a solution for all n, so $Y(x,t) = \sum_{n=1}^{\infty} Y_n$ is also a solution, and

$Y(x,t) = \sum_{n=1}^{\infty} \sin k_n x \,(A_n \cos \omega_n t + B_n \sin \omega_n t)$ is general soln ✓

(b) String motionless at $t=0$ implies

$\frac{dY(x,t)}{dt} = 0$ ✓

$\frac{dY(x,t)}{dt} = \sum_{n=1}^{\infty} \omega_n \sin k_n x \,(-A_n \sin \omega_n t + B_n \cos \omega_n t)$ ✓

$\frac{dY(x,t)}{dt} = 0$ at $t=0 \Rightarrow B_n = 0$ ✓

This is almost impossible to read. you should lay your work out more clearly

ie $Y(x,t) = \sum_{n=1}^{\infty} A_n \sin k_n x \cos \omega_n t$ ✓

To determine the A_n, I would need the initial string profile. Suppose this is $Y_0(x)$, ie

$Y_0(x) = \sum_{n=1}^{\infty} A_n \sin k_n x$

$Y_0(x) \sin k_m x = \sum_{n=1}^{\infty} A_n \sin^2 k_n x$

$\int_0^L Y_0(x) \sin k_m x \, dx = \sum_{n=1}^{\infty} \int_0^L A_n \sin k_n x \sin k_m x \, dx$

$\int_0^L Y_0(x) \sin k_n x \, dx = \frac{A_n L}{2}$ $\quad n = m$

$= 0$ $\quad n \neq m$

P.T.O