d) The probability of finding the particle in the interval $[x, x+\Delta x]$ is $|\psi(x)|^2 \Delta x$, approximately.

ie $\Delta P \simeq |\psi(x)|^2 \Delta x$

Letting $\Delta P, \Delta x \to 0$, we can convert to an integral

$$\int dP = \int_{-\infty}^{\infty} |\psi(x)|^2 dx$$

ie $$1 = \int_{-\infty}^{\infty} N^2 \left(\frac{4x^2}{a^2}\right) \exp\left(\frac{-x^2}{a^2}\right) dx$$

$$= 4N^2 \int \left(\frac{x^2}{a^2}\right) \exp\left(\frac{-x^2}{a^2}\right) dx$$

sub $y = \frac{x}{a}$, so $dx = a\,dy$

$$1 = 4N^2 a \int_{-\infty}^{\infty} y^2 \exp(-y^2)$$

$$= 4N^2 a \frac{\sqrt{\pi}}{2}$$ Using the given integral

$$N^2 = \frac{1}{2a\sqrt{\pi}} \Rightarrow N = \frac{1}{\cancel{2}(2a\sqrt{\pi})^{1/2}}$$

The forbidden region is the union of the open intervals $(-\infty, -a\sqrt{3})$, $(a\sqrt{3}, \infty)$. Then the probability of finding the particle in the forbidden region is $\int_{-\infty}^{-a\sqrt{3}} |\psi(x)|^2 dx + \int_{a\sqrt{3}}^{\infty} |\psi(x)|^2 dx$

$$= 1 - \int_{-a\sqrt{3}}^{a\sqrt{3}} |\psi(x)|^2 dx$$ by properties of integration

$$= 1 - 2\int_{0}^{a\sqrt{3}} |\psi(x)|^2 dx$$ by symmetry of $|\psi(x)|^2$

$$= 1 - 2\int_{0}^{a\sqrt{3}} N^2 \left(\frac{4x^2}{a^2}\right) \exp\left(-\frac{x^2}{a^2}\right) dx$$

error in N

$$= 1 - 2\int_{0}^{a\sqrt{3}} \frac{1}{\cancel{4}a\sqrt{\pi}} \times 4\left(\frac{x^2}{a^2}\right) \exp\left(-\frac{x^2}{a^2}\right) dx$$

$$= 1 - \frac{\cancel{4}}{a\sqrt{\pi}} \int_{0}^{a\sqrt{3}} \left(\frac{x^2}{a^2}\right) \exp\left(\frac{-x^2}{a^2}\right) dx$$

Put $y = \frac{x}{a}$ then $dx = a\,dy$, the above becomes