where $\hat{\mathrm{H}}$ is the hamiltonian of Section 4.3 of Unit 15 and $E_{\mathrm{R}} = 13.61\,\mathrm{eV}$ is the Rydberg energy. Use this result to make the best variational estimate of the energy of the lowest state of ortho-helium, and select the item from the key that is closest to your answer.

KEY for Q51

A	$-57.8\,\mathrm{eV}$	E	$-59.4\,\mathrm{eV}$
B	$-58.2\,\mathrm{eV}$	F	$-59.8\,\mathrm{eV}$
C	$-58.6\,\mathrm{eV}$	G	$-60.2\,\mathrm{eV}$
D	$-59.0\,\mathrm{eV}$	H	$-60.6\,\mathrm{eV}$

Q52 Consider the hamiltonian operator, $\hat{\mathrm{H}}$, for the helium atom where the nucleus (located at the origin) is assumed to have effectively infinite mass and where the two electrons interact with the nucleus and with each other via the Coulomb potential. Then

$\hat{\mathrm{H}}$ = (kinetic energy operator for the electrons) + (Coulomb interaction operator for each electron with the nucleus) + (Coulomb interaction operator for the two electrons with each other).

The electrons, each of charge $-e$ (where $e > 0$), have positions $\mathbf{r}_1$ and $\mathbf{r}_2$. Select from the key the term representing the interaction of the electrons with each other.

KEY for Q52

A $\dfrac{e^2}{4\pi\varepsilon_0(|\mathbf{r}_1| - |\mathbf{r}_2|)}$

B $-\dfrac{e^2}{4\pi\varepsilon_0|\mathbf{r}_1|} - \dfrac{e^2}{4\pi\varepsilon_0|\mathbf{r}_2|}$

C $\dfrac{2e^2}{4\pi\varepsilon_0|\mathbf{r}_1|} + \dfrac{2e^2}{4\pi\varepsilon_0|\mathbf{r}_2|}$

D $-\dfrac{2e^2}{4\pi\varepsilon_0|\mathbf{r}_1|} - \dfrac{2e^2}{4\pi\varepsilon_0|\mathbf{r}_2|}$

E $\dfrac{e^2}{4\pi\varepsilon_0|\mathbf{r}_1 - \mathbf{r}_2|}$

F $\dfrac{2e^2}{4\pi\varepsilon_0|\mathbf{r}_1 - \mathbf{r}_2|}$

G $-\dfrac{2e^2}{4\pi\varepsilon_0|\mathbf{r}_1 - \mathbf{r}_2|}$

Q53 Suppose the hydrogen-like energy eigenstates for a single electron moving under the influence of a nuclear charge $2e$ are denoted by $\psi_1(\mathbf{r})$, $\psi_2(\mathbf{r})$, etc., where ψ_1 represents the ground state, and so forth. In this question, you are asked to choose to zero order, a triplet state of helium with one electron in the ground state and the other in the second excited state. (To zero order, one neglects the Coulomb interaction between the electrons.) The full zero-order state is represented by a spatial wave function multiplied by a spin wave function formed from single-particle spinors α and β. Choose from items A–D in the key the correct spatial part and from items E–H the correct spin part.

KEY for Q53

A $\psi_1(\mathbf{r}_1)\,\psi_3(\mathbf{r}_2)$

B $\dfrac{1}{\sqrt{2}}(\psi_1(\mathbf{r}_1)\,\psi_3(\mathbf{r}_2) + \psi_1(\mathbf{r}_2)\,\psi_3(\mathbf{r}_1))$

C $\dfrac{1}{\sqrt{2}}(\psi_1(\mathbf{r}_1)\,\psi_3(\mathbf{r}_2) - \psi_1(\mathbf{r}_2)\,\psi_3(\mathbf{r}_1))$

D $(\psi_1(\mathbf{r}_1)\,\psi_3(\mathbf{r}_2) - \psi_1(\mathbf{r}_2)\,\psi_3(\mathbf{r}_1))$

E $\alpha(1)\,\beta(2)$

F $\dfrac{1}{\sqrt{2}}[\alpha(1)\,\beta(2) + \alpha(2)\,\beta(1)]$

G $\beta(1)\,\alpha(2)$

H $\dfrac{1}{\sqrt{2}}[\alpha(1)\,\beta(2) - \alpha(2)\,\beta(1)]$

Q54 Ignoring all interactions between the two electrons, what is the degeneracy of helium in the state with one electron in the hydrogen-like ground state ($n = 1$) and the other electron in the third excited state ($n = 4$)?

KEY for Q54

A	2	E	28
B	8	F	36
C	18	G	64
D	22	H	128

Q55 The results of one particular option in the key, taken from experiments mentioned in Unit 1 and elsewhere, become intelligible in the light of the last section of Unit 15. Which one is it?

KEY for Q55

A Davison–Germer and G. P. Thomson experiments

B Compton effect experiment

C Millikan's photoelectric effect measurements

D Moseley's X-ray measurements

E Stern–Gerlach experiment

F Geiger–Marsden experiment

G Zeeman effect experiments

Q56 to Q66 mainly concern Unit 16.

[Hint: it will be helpful if you realize that the factor $\frac{1}{2}\left(\dfrac{e^2}{4\pi\varepsilon_0 m^2 c^2}\right)\dfrac{\hbar^2}{a_0^2} = \alpha^2 E_{\mathrm{R}}$, and that $\dfrac{E_{\mathrm{R}}}{2mc^2} = \frac{1}{4}\alpha^2$.]

Q56 to Q60 share the same key and concern the first-order contributions of the spin–orbit perturbation, $\delta\hat{\mathbf{H}}_{\text{spin-orbit}}$, the relativistic perturbation, $\delta\hat{\mathbf{H}}_{\text{rel}}$, the Darwin term, $\delta\hat{\mathbf{H}}_{\text{Darwin}}$, and their sum

$$\delta\hat{\mathbf{H}}_{\text{total}} = \delta\hat{\mathbf{H}}_{\text{spin-orbit}} + \delta\hat{\mathbf{H}}_{\text{rel}} + \delta\hat{\mathbf{H}}_{\text{Darwin}}$$

to the energies of the $2^2\mathrm{S}_{1/2}$ and $2^2\mathrm{P}_{3/2}$ states of the hydrogen atom. In each case, you are to select from the key the item that gives the contribution in question.