with $\hat{p}_x$, and it's the same for $\hat{K}$. Finally $\hat{L} = -1$ (see Q9 answer) and therefore commutes with any operator. Only $\hat{V}$ and $\hat{H}$ fail to commute with $\hat{p}_x$.

Q11 $\boxed{F}$ For $x < -L, V(x) = 9h^2/32mL^2$, and so using the result of Unit 6 Exercise 4

$$\begin{aligned} k_A &= (2m(E-V))^{1/2}/\hbar \\ &= (2m(25-9)h^2/32mL^2)^{1/2}/\hbar = h/\hbar L. \end{aligned}$$

Thus $k_A = 2\pi/L$. (See FT Sec. 9-4.)

Q12 $\boxed{H}$ For $|x| \leqslant L, V(x) = 0$, and so

$$\begin{aligned} k_B &= (2mE)^{1/2}/\hbar \\ &= (2m \times 25h^2/32mL^2)^{1/2}/\hbar \\ &= 5h/4L\hbar = 5\pi/2L. \end{aligned}$$

(See FT Sec. 9-4.)

Q13 $\boxed{F}$ For $x > L, V(x)$ is the same as in Q11. Hence $k_C = k_A = 2\pi/L$.

Q14 $\boxed{A,G}$ Use continuity of ψ at the two boundaries using values of k_A, k_B and k_C from Q11–Q13. (See FT Sec. 9-4.) Thus at $x = -L$ we have

$$\begin{aligned} &A_1 e^{i2\pi(-L)/L} + A_2 e^{-i2\pi(-L)/L} \\ &= B_1 e^{i5\pi(-L)/2L} + B_2 e^{-i5\pi(-L)/2L}. \end{aligned}$$

This gives $A_1 + A_2 = -iB_1 + iB_2$ or $B_1 - B_2 = i(A_1 + A_2)$.

Now use continuity of ψ at $x = L$ (See FT Sec. 9-4.)

$$\begin{aligned} &B_1 e^{i5\pi L/2L} + B_2 e^{-i5\pi L/2L} \\ &= C_1 e^{i2\pi L/L} + C_2 e^{-i2\pi L/L}. \end{aligned}$$

This gives $B_1 i + B_2(-i) = C_1 + C_2$ or $B_1 - B_2 = -i(C_1 + C_2)$.

Q15 $\boxed{B,H}$ Use continuity of ψ' at the two boundaries. At $x = L$

$$\begin{aligned} &B_1(i5\pi/2L)e^{i5\pi/2} + B_2(-i5\pi/2L)e^{-i5\pi/2} \\ &= C_1(i2\pi/L)e^{i2\pi} + C_2(-i2\pi/L)e^{-i2\pi}. \end{aligned}$$

This gives

$$(5/2)\,(B_1 i + B_2 i) = 2\,(C_1 - C_2)$$

or

$$B_1 + B_2 = -(4/5)i\,(C_1 - C_2).$$

At $x = -L$,

$$\begin{aligned} &A_1(i2\pi/L)e^{-i2\pi} + A_2(-i2\pi/L)e^{i2\pi} \\ &= B_1(i5\pi/2L)e^{-i5\pi/2} + B_2(-i5\pi/2L)e^{i5\pi/2}. \end{aligned}$$

This gives

$$2\,(A_1 - A_2) = (5/2)\,(B_1(-i) - B_2 i)$$

or

$$B_1 + B_2 = (4/5)i\,(A_1 - A_2).$$

Q16 $\boxed{B}$ The probability current is (FT Eq. 9.8b)

$$\begin{aligned} J &= (-i\hbar/2m)(\psi^*\psi' - \psi\psi^{*\prime}) \\ &= (-i\hbar/2m)[(B_1^* e^{-ik_B x} + B_2^* e^{ik_B x}) \\ &\quad (ik_B B_1 e^{ik_B x} - ik_B B_2 e^{-ik_B x}) \\ &\quad - (B_1 e^{ik_B x} + B_2 e^{-ik_B x}) \\ &\quad (-ik_B B_1^* e^{-ik_B x} + ik_B B_2^* e^{ik_B x})] \\ &= (\hbar k_B/m)\left(|B_1|^2 - |B_2|^2\right). \end{aligned}$$

Q17 $\boxed{B}$ The steady-state probability current must be the same in all three regions because there are no sources or sinks of particles, i.e. particles are neither created nor destroyed, merely reflected or transmitted. The answer to Q17 is therefore equal to the answer to Q16.

Q18 $\boxed{E}$ Use $\gamma = gq/2M$ (Unit 8, page 8). Then $2.67 \times 10^8\,\text{s}^{-1}\,\text{T}^{-1} = g \times 1.602 \times 10^{-19}\,\text{C}/2 \times 1.673 \times 10^{-27}\,\text{kg}$ which gives the gyromagnetic ratio $g = 5.58$.

Q19 $\boxed{D}$ The energy of the transition from parallel, to antiparallel, is $E = 2|\boldsymbol{\mu} \cdot \mathbf{B}| = 2|\mu_z|B$ where μ_z is the component of the proton's magnetic moment along the magnetic field direction and B is the magnitude of the field. (See Unit 8 Sec 1.2.) $|\mu_z| = \gamma\hbar/2$ where $\hbar/2$ is the magnitude of the z-component of angular momentum (spin) associated with the proton. (See Exercise 15 of Unit 8.) Thus we have $E = (2\gamma\hbar/2)B = 2.67 \times 10^8\,\text{s}^{-1}\,\text{T}^{-1} \times 1.055 \times 10^{-34}\,\text{J s} \times 1\,\text{T} = 2.817 \times 10^{-26}\,\text{J}$. The associated frequency is $f = E/h = 4.25 \times 10^7$ Hz.

Q20 $\boxed{A,B}$ $\hat{L}_z = \dfrac{\hbar}{i}\dfrac{\partial}{\partial\phi}$ operates on functions $f(r, \phi, z)$. For $f = x = r\cos\phi$,

$$\hat{L}_z x = \frac{\hbar}{i} r(-\sin\phi).$$

Therefore x is not an eigenfunction. Similarly

$$\begin{aligned} \hat{L}_z xy &= \frac{\hbar}{i}\frac{\partial}{\partial\phi}\left(r^2 \sin\phi\cos\phi\right) \\ &= \frac{\hbar}{i} r^2 \cos 2\phi \end{aligned}$$

and so xy is not an eigenfunction. The other options are all eigenfunctions:

$$\hat{L}_z z = \frac{\hbar}{i}\frac{\partial z}{\partial\phi} = 0 \qquad \text{(eigenvalue} = 0\text{)},$$

$$\begin{aligned} \hat{L}_z\left(x^2 + y^2 + z^2\right) &= \frac{\hbar}{i}\frac{\partial}{\partial\phi}\left(r^2 + z^2\right) \\ &= 0 \qquad \text{(eigenvalue} = 0\text{)}, \end{aligned}$$

$$\begin{aligned} \hat{L}_z(x + iy) &= \frac{\hbar}{i}\frac{\partial}{\partial\phi}\left(re^{i\phi}\right) \\ &= \hbar re^{i\phi} \qquad \text{(eigenvalue} = \hbar\text{)} \end{aligned}$$

and similarly

$$\hat{L}_z(x - iy) = -\hbar(x - iy) \qquad \text{(eigenvalue} = -\hbar\text{)}.$$