

**Q33** Select *one* option to describe the spectrum of the momentum operator for a free particle moving in one dimension.

**Q34** Select *one* option to describe the spectrum of the energy operator for a hydrogen atom.

**Q35** Select *one* option to describe the spectrum of the energy operator for a particle in a finite one-dimensional square well.

KEY for Q32 to Q35

- A A single discrete real number
- B A finite set of several discrete real numbers
- C An infinite set of discrete real numbers
- D All the complex numbers
- E All the real numbers
- F All the real numbers greater than or equal to a certain real number
- G All the real numbers greater than or equal to a certain real number  $C$ , together with a finite set of discrete real numbers less than  $C$
- H All the real numbers greater than or equal to a certain real number  $C$ , together with an infinite set of discrete real numbers less than  $C$

#### PART D (Units 12-14)

*Q36 and Q37 mainly concern Unit 12.*

Q36 concerns a rotating molecule in an orbital angular momentum state represented by the normalized wave function

$$\psi(\theta, \phi) = aY_{1,1}(\theta, \phi) + bY_{1,0}(\theta, \phi) + cY_{1,-1}(\theta, \phi)$$

where  $a$ ,  $b$  and  $c$  are complex constants, and  $Y_{1,1}(\theta, \phi)$ ,  $Y_{1,0}(\theta, \phi)$  and  $Y_{1,-1}(\theta, \phi)$  are the spherical harmonics in Table 11-2 on FT p. 485.

**Q36** What is the probability of obtaining the value  $2\hbar^2$  when the square of the magnitude of the orbital angular momentum of the molecule is measured?

KEY for Q36

- |           |                           |
|-----------|---------------------------|
| A 0       | E $ c ^2$                 |
| B 1       | F $ a ^2 +  c ^2$         |
| C $ a ^2$ | G $ a ^2 +  c ^2 + 2ac^*$ |
| D $ b ^2$ | H $\frac{1}{2} a + c ^2$  |

Q37 refers to the spinor of functions

$$\psi = \frac{1}{\sqrt{2}}[Y_{1,1} - Y_{1,0}]^T$$

where  $Y_{1,1}$  and  $Y_{1,0}$  are spherical harmonics.

**Q37** What spinor of functions is equal to  $\hat{L}_z\psi$ , where  $\hat{L}_z$  is the operator representing the  $z$ -component of the orbital angular momentum?

KEY for Q37

- A  $(\hbar/\sqrt{2})[Y_{1,0} \ Y_{1,1}]^T$
- B  $(\hbar/\sqrt{2})[Y_{1,1} \ 0]^T$
- C  $(\hbar/2)[Y_{1,1} \ 0]^T$
- D  $\hbar[-Y_{1,1} \ Y_{1,0}]^T$
- E  $(\hbar/2\sqrt{2})[Y_{1,1} \ -Y_{1,0}]^T$
- F  $(\hbar/2\sqrt{2})[Y_{1,1} \ Y_{1,0}]^T$
- G  $\hbar[Y_{1,0} \ Y_{1,1}]^T$
- H  $\hbar[-Y_{1,0} \ -Y_{1,1}]^T$

*Q38 to Q42 mainly concern Unit 13*

Q38 to Q42 concern the stationary-state wave functions  $\psi_{n,l,m}(r, \theta, \phi)$  of the Coulomb model of the hydrogen atom.

**Q38** Select from the key the *three* items that correspond to *valid* combinations of values of the quantum numbers  $n, l, m$ .

KEY for Q38

- |                   |                        |
|-------------------|------------------------|
| A $\psi_{2,2,-1}$ | E $\psi_{3,3,-2}$      |
| B $\psi_{3,2,-3}$ | F $\psi_{3,2,1}$       |
| C $\psi_{2,0,1}$  | G $\psi_{137,136,136}$ |
| D $\psi_{2,0,0}$  | H $\psi_{137,137,0}$   |

Q39 to Q42 share the same key, which consists of valid hydrogen wave functions. In each case, you are to select from the key the wave functions that have the specified property or properties.

**Q39** Which *three* wave functions represent states of the same energy?

**Q40** Which *two* wave functions represent states of the same energy *and* have the same  $\theta$  dependence?

**Q41** Which *two* wave functions have precisely *one* node in their  $r$  dependence for finite positive  $r$ ?

**Q42** Which *two* wave functions have the same  $\phi$  dependence *and* the same  $\theta$  dependence?

KEY for Q39 to Q42

- |                   |                   |
|-------------------|-------------------|
| A $\psi_{7,6,-3}$ | E $\psi_{4,1,0}$  |
| B $\psi_{6,5,3}$  | F $\psi_{4,1,-1}$ |
| C $\psi_{5,3,2}$  | G $\psi_{3,1,1}$  |
| D $\psi_{4,1,1}$  | H $\psi_{2,0,0}$  |