PART A (Units 3 and 4)

Q1 to Q4 mainly concern Unit 3.

Q1 concerns solutions to the one-dimensional Schrödinger equation.

Q1 Let $\psi(x)$ be an *acceptable* solution to the time-independent Schrödinger equation with a potential energy function $V(x)$ that is finite and continuous everywhere. The key contains precisely *two* statements that are not necessarily true for such a solution. Select the *two* options that are *not* necessarily true.

KEY for Q1

A $\psi(x)$ is continuous everywhere. ✓
B $\psi'(x)$ is continuous everywhere. ✓
C $\psi''(x)$ is continuous everywhere. ✗
D $|\psi(x)|$ is an even function of x. ✗
E $\psi(x)$ is finite everywhere. ✓
F $\psi'(x)$ is finite everywhere. ✓
G $\psi''(x)$ is finite everywhere. ✓
H $|\psi(x)|$ vanishes as $|x| \to \infty$. ✓

Q2 to Q4 concern a particle of mass m confined within an infinite square well with potential energy function given by

$$V(x) = \begin{cases} 0 & \text{for } 0 \leqslant x \leqslant L \\ \infty & \text{for } x > L \text{ and } x < 0. \end{cases}$$

Q2 and Q3 share the same key.

Q2 Suppose that the particle is in the ground state. Select the option that gives the probability for finding the particle in the interval $[L/3, 2L/3]$.

Q3 Suppose that the particle is in the first excited state. Select the option that gives the probability for finding the particle in the interval $[L/6, 5L/6]$.

KEY for Q2 and Q3

A	$\frac{1}{2}$	E	$\frac{1}{3} + \sqrt{3}/(2\pi)$
B	$\frac{1}{3}$	F	$\frac{1}{3} - \sqrt{3}/(2\pi)$
C	$\frac{2}{3}$	G	$\frac{2}{3} + \sqrt{3}/(4\pi)$
D	$\frac{1}{6}$	H	$\frac{2}{3} - \sqrt{3}/(4\pi)$

Q4 Let $\lambda_0 \equiv 8mL^2c/h$. Select the option that is the wavelength of the photon emitted when the particle makes a transition from the third excited state to the first excited state of the infinite square well.

KEY for Q4

A	$2\lambda_0$	E	$4\lambda_0$
B	$\lambda_0/3$	F	$\lambda_0/9$
C	$3\lambda_0$	G	$5\lambda_0$
D	$\lambda_0/5$	H	$\lambda_0/12$

Q5 and Q6 mainly concern Unit 4.

Q5 and Q6 concern a particle of mass m in a harmonic oscillator potential well with potential energy function $V(x) = \frac{1}{2}m\omega^2x^2$.

Q5 Select the *two* options that are *false* statements.

KEY for Q5

A If the particle is in a stationary state, then the modulus of its wave function is time-*independent*. ✓
B If the particle is in a non-stationary state, then the modulus of its wave function is time-*dependent*. ✓
C If the particle is in a stationary state, then *every* measurement of energy will yield an odd multiple of the ground-state energy. ✓
D If the particle is in a non-stationary state, then *not* every measurement of energy will yield an odd multiple of the ground-state energy. ✗ ?
E If the particle is in a stationary state, then the probability of finding it at negative values of x at a given time is *equal* to that for finding it at positive values of x at that time. ✓
F A particle in a stationary state, represented by $\psi_n(x)$, is more likely to be found in the region $x \geqslant 0$ when n is large than when $n = 1$. ✗
G If the particle is in a stationary state, then the first derivative of its wave function with respect to x *necessarily* vanishes for at least one value of x in the range $\infty > x \geqslant 0$. ✓
H If the particle is in a stationary state, then the second derivative of its wave function with respect to x *necessarily* vanishes for at least one value of x in the range $\infty > x \geqslant 0$. ✓

Q6 It is found that a certain stationary state of the oscillator has a probability density with precisely 7 local maxima. Select the option that gives the energy of this state.

KEY for Q6

A	$3\hbar\omega/2$	E	$11\hbar\omega/2$
B	$5\hbar\omega/2$	F	$13\hbar\omega/2$
C	$7\hbar\omega/2$	G	$15\hbar\omega/2$
D	$9\hbar\omega/2$	H	$17\hbar\omega/2$

PART B (Units 5–8)

Q7 to Q10 mainly concern Unit 5.

Q7 and Q8 concern a particle of mass m in a harmonic oscillator potential well with potential energy function $V(x) = \frac{1}{2}m\omega^2x^2$.

Q7 Consider a wave packet $\Psi(x,t)$ formed from the first excited-state wave function $\psi_1(x)$ and the second excited-state wave function $\psi_2(x)$. At time $t = 0$, the wave function is

$$\Psi(x, 0) = 0.8\psi_1(x) + 0.6\psi_2(x).$$