Q30 $\boxed{\text{B,C,G}}$ See Section 3 of Unit 10, especially the second row of Table 4 on p. 33.

Q31 $\boxed{\text{A,D,H}}$ See Section 3 of Unit 10, especially the second row of Table 4 on p. 33.

Q32 $\boxed{\text{E}}$ See Section 1.4 of Unit 11.

Q33 $\boxed{\text{E}}$ See Section 2.3 of Unit 11.

Q34 $\boxed{\text{H}}$ There are two parts to the spectrum. An infinite number of discrete energies $(E_n = (-13.6/n^2)$ eV for $n = 1, 2, 3 \ldots)$ and the continuum of positive energies. Thus the answer is option H. (C is usually taken to be 0.)

Q35 $\boxed{\text{G}}$ See Section 3 of Unit 11.

Q36 $\boxed{\text{B}}$ The given wavefunction is a linear superposition of $Y_{l=1,m} (m = -1, 0, +1)$. It is therefore an eigenfunction of $\hat{L}^2$ with $l = 1$ and eigenvalue $l(l+1)\hbar^2 = 2\hbar^2$. (See FT p. 483 to end of Section 11.2.)

Q37 $\boxed{\text{B}}$ The spherical harmonics $Y_{1,1}$ and $Y_{1,0}$ are eigenfunctions of $\hat{L}_z$ with eigenvalues $\hbar$ and 0 respectively. Hence

$$\begin{aligned}\hat{L}_z\psi &= \frac{1}{\sqrt{2}}[\hbar Y_{1,1} \quad 0Y_{1,0}]^T \\ &= \frac{\hbar}{\sqrt{2}}[Y_{1,1} \quad 0]^T\end{aligned}$$

Q38 $\boxed{\text{D,F,G}}$ Options A, E and H violate $l \leqslant n-1$; options B, C violate $|m| \leqslant l$.

Q39 $\boxed{\text{D,E,F}}$ The energy is $-E_R/n^2$; only options D,E,F share a value of n.

Q40 $\boxed{\text{D,F}}$ The θ dependences of $Y_{1,1}$ and $Y_{1,-1}$ are the same, but that of $Y_{1,0}$ is different. (The fact that $Y_{1,-1} = -Y_{1,1}$ does not amount to different θ dependence.)

Q41 $\boxed{\text{G,H}}$ The number of radial nodes is $n - l - 1$.

Q42 $\boxed{\text{D,G}}$ For the same θ and ϕ dependence, $l_1 = l_2, m_1 = m_2$.

Q43 $\boxed{\text{D,H}}$ For example, to show D, write down $\hat{\mathbf{H}}\phi = \psi$ and the terms which are first order in k give

$$\mathbf{H}^{(0)}\phi^{(1)} + \mathbf{H}^{(1)}\phi^{(0)} = \psi^{(1)}.$$

Taking the inner product of this equation with $\phi^{(0)}$ gives

$$\left(\phi^{(0)}, \mathbf{H}^{(0)}\phi^{(1)}\right) + \left(\phi^{(0)}, \mathbf{H}^{(1)}\phi^{(0)}\right) = \left(\phi^{(0)}, \psi^{(1)}\right).$$

Since $\mathbf{H}^{(0)}$ is hermitian this is equivalent to

$$\left(\mathbf{H}^{(0)}\phi^{(0)}, \phi^{(1)}\right) + \left(\phi^{(0)}, \mathbf{H}^{(1)}\phi^{(0)}\right) = \left(\phi^{(0)}, \psi^{(1)}\right).$$

We now use $\hat{\mathbf{H}}^{(0)}\phi^{(0)} = \psi^{(0)}$ to give

$$\left(\psi^{(0)}, \phi^{(1)}\right) + \left(\phi^{(0)}, \mathbf{H}^{(1)}\phi^{(0)}\right) = \left(\phi^{(0)}, \psi^{(1)}\right),$$

or

$$\begin{aligned}\left(\phi^{(0)}, \mathbf{H}^{(1)}\phi^{(0)}\right) &= \left(\phi^{(0)}, \psi^{(1)}\right) - \left(\psi^{(0)}, \phi^{(1)}\right) \\ &= \left(\phi^{(0)}, \psi^{(1)}\right) - \left(\phi^{(1)}, \psi^{(0)}\right)^*,\end{aligned}$$

which is option D. Option H can similarly be obtained by considering the terms in $\hat{\mathbf{H}}\phi = \psi$ which are second order in k and again taking the inner product of the resulting equation with $\phi^{(0)}$.

Q44 $\boxed{\text{C,D}}$

$$\begin{aligned}&\left(\frac{1}{\sqrt{2}}\left(\psi_m^{(0)} + \psi_n^{(0)}\right), \delta\hat{\mathbf{H}}\frac{1}{\sqrt{2}}\left(\psi_m^{(0)} - \psi_n^{(0)}\right)\right) \\ &= \tfrac{1}{2}\{(\psi_m^{(0)}, \delta\hat{\mathbf{H}}\psi_m^{(0)}) - (\psi_m^{(0)}, \delta\hat{\mathbf{H}}\psi_n^{(0)}) \\ &\quad + (\psi_n^{(0)}, \delta\hat{\mathbf{H}}\psi_m^{(0)}) - (\psi_n^{(0)}, \delta\hat{\mathbf{H}}\psi_n^{(0)})\} \\ &= \tfrac{1}{2}(-0.6A - 0.8A + 0.8A + 0.6A) = 0,\end{aligned}$$

as required by Eq. 53 of Unit 14. No other two options satisfy this condition.

Q45 $\boxed{\text{A,H}}$ The first-order corrections for the energy are given by Eq. 14 of Unit 14, i.e.

$$\begin{aligned}&\tfrac{1}{2}\left(\left(\psi_m^{(0)} + \psi_n^{(0)}\right), \delta\hat{\mathbf{H}}\left(\psi_m^{(0)} + \psi_n^{(0)}\right)\right) \\ &= \tfrac{1}{2}(-0.6A - 0.6A + 0.8A + 0.8A) = 0.2A,\end{aligned}$$

and

$$\begin{aligned}&\tfrac{1}{2}\left(\left(\psi_m^{(0)} - \psi_n^{(0)}\right), \delta\hat{\mathbf{H}}\left(\psi_m^{(0)} - \psi_n^{(0)}\right)\right) \\ &= \tfrac{1}{2}(-0.6A - 0.6A - 0.8A - 0.8A) = -1.4A.\end{aligned}$$

Q46 $\boxed{\text{A}}$ All four particles can have $n = l = m = 1$.

Q47 $\boxed{\text{C}}$ Two particles have $n = l = m = 1$, two can have $n = 1$ and $l = 2, m = 1$ or $l = 1, m = 2$. (Only *two* fermions can have same n, l, m.)

Q48 $\boxed{\text{A}}$ All four particles may have $n = l = m = 1$.

Q49 $\boxed{\text{B}}$ Three have $n = l = m = 1$, one has for example $n = m = 1, l = 2$.

Q50 $\boxed{\text{D}}$ To obtain the first excited states of the system we could excite one fermion from the $n = l = m = 1$ state to one of the vacant single particle first excited states $n = l = 1, m = 2$ and $n = m = 1, l = 2$. So the energy of the first excited state of the system is $E_0 + 3 \times \frac{3}{2}E_0 = \frac{11}{2}$. Alternatively we could excite one of the fermions from the single particle first excited states to the single particle second excited state $n = 1, l = m = 2$. This also has energy