SM355, CMA51 1999 Model answers

Q1 $\boxed{\text{G}}$ 1 mole of any compound contains 6.022×10^{23} molecules (see Frame 3 on p6 of Unit 1). 1 mole of nitrogen gas (N_2) has a mass of approximately 0.028 kg where the relative atomic mass is taken to be 2×14. The number of molecules in 1 kg is therefore $6.022 \times 10^{23}/0.028 = 2.15 \times 10^{25}$.

Q2 $\boxed{\text{C}}$ Density = mass/volume. The mass of one mole of nitrogen gas is 0.028 kg and the volume is given as 22.4×10^{-3} m^3.
$\therefore$ density $= 0.028\,\text{kg}/22.4 \times 10^{-3}\,\text{m}^3 = 1.25\,\text{kg m}^{-3}$.

Q3 $\boxed{\text{C}}$ The de Broglie wavelength is $\lambda = h/p$ (Sec.3.1 of Unit 1, FT 2-3). Electrons accelerated from rest through 60 volts have a kinetic energy of $E = 60$ eV. This is a low (i.e. non-relativistic) energy and so the momentum p is given by $(2mE)^{1/2}$ and so

$$\lambda = h/p = \frac{h}{(2mE)^{1/2}} = \frac{6.626 \times 10^{-34}}{(2 \times 9.109 \times 10^{-31} \times 60 \times 1.602 \times 10^{-19})^{1/2}}\,\text{m}$$
$$= 1.583 \times 10^{-10}\,\text{m}.$$

Q4 $\boxed{\text{A}}$ Low energy electrons are reflected from the surface of a crystal (Davisson-Germer experiments. See FT 2-4) at angle ϕ given by $\lambda = D \sin\phi$ where D is the required spacing. Hence $D = 1.583 \times 10^{-10}\,\text{m}/\sin 30^\circ = 3.17 \times 10^{-10}\,\text{m}$.

Q5 $\boxed{\text{F}}$ The wavelengths of spectral lines emitted by hydrogen-like systems according to the Bohr theory are found from (Sec.2-3 of Unit 1, FT Eqs.1-24, 1-25)

$$\frac{1}{\lambda} = R_\text{H} Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where n_i and n_f are positive integers with $n_i > n_f$. For H, $Z = 1$. Trying values of n_f and n_i, starting with $n_f = 1, n_i = 2, 3$ etc, then $n_f = 2, n_i = 3, 4$ etc, you can see that the only possible option is F for which $n_f = 2, n_i = 3$. Thus $\frac{1}{\lambda} = R_\text{H}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R_\text{H}\frac{5}{36}$ and so $\lambda = 36/5R_\text{H}$.

Q6 $\boxed{\text{C,F}}$ As for Q5 but with $Z = 2$ for He^+. Trying values of n_f and n_i, you find $n_f = 1, n_i = 2$ yields $\frac{1}{\lambda} = R_\text{H}4\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ which gives $\lambda = 1/3R_\text{H}$, option C. The next success is with $n_f = 4, n_i = 6$ which gives $\frac{1}{\lambda} = R_\text{H}4\left(\frac{1}{4^2} - \frac{1}{6^2}\right) = \frac{5}{36}R_\text{H}$ which corresponds to option F. (See last paragraph of FT 1.8.)

Q7 $\boxed{\text{D}}$ We have $L_1 = 0.529$ Å$= 0.529 \times 10^{-10}$ m (FT Eq.1-23b). L_2 is seen from FT Fig.1-14 to be just greater than 0.7 Å$= 0.7 \times 10^{-10}$ m (a more precise value is given on FT p.38). L_3 is found from using $\lambda = h/p$ (FT Eq.2-1) with $p = (2mK)^{1/2}$ for low energies. (See FT Eq.2.9.) Thus

$$L_3 = \lambda = \frac{h}{(2mK)^{1/2}} = \frac{6.626 \times 10^{-34}}{(2 \times 10^{-3} \times (28/6.022 \times 10^{23}) \times 0.025 \times 1.602 \times 10^{-19})^{1/2}}\,\text{m}$$

where the mass of a nitrogen molecule m has been found by using $Nm = 10^{-3}A$ (see bottom of Frame 3 on p.6 of Unit 1). Thus we find $L_3 = 3.43 \times 10^{-11}$ m. L_4 is found from FT Eq.2.1,

$$L_4 = \lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{0.15 \times 40}\,\text{m} = 1.10 \times 10^{-34}\,\text{m}.$$

Finally to find L_5 we have to use the relativistic formula for de Broglie wavelengths, FT Eq.2-10, or FT Eq.2-11 which is a good approximation for $K = 10^{21}$ eV. Thus

$$L_5 \approx \frac{hc}{K} = \frac{6.626 \times 10^{-34} \times 2.998 \times 10^8}{10^{21} \times 1.602 \times 10^{-19}}\,\text{m} = 1.24 \times 10^{-27}\,\text{m}.$$

Thus the correct ordering is $L_2 > L_1 > L_3 > L_5 > L_4$ which is option D.