v). $\rho^r = aT^4$

$\therefore \rho_d^r = a T_d^4$

$= 7.5641 \times 10^{-16} J m^{-3} K^{-4} \times (3000K)^4$

$= 0.061269 J/m^3$

$= 0.06 J/m^3$ ~~to~~ 2 decimal places.

Now since radiation and matter have interacted negligibly since decoupling we can assume

$$\rho^r = \rho_0^r \left(\frac{R(t_0)}{R(t)}\right)^4 \quad \text{for } t \geq t_d$$

$$\rho_d^r = \rho_0^r \left(\frac{R(t_0)}{R(t_d)}\right)^4$$

$$\therefore \frac{R(t_0)}{R(t_d)} = \sqrt[4]{\frac{\rho_d^r}{\rho_0^r}}$$

$$\frac{R(t_0)}{R(t_d)} = \sqrt[4]{\frac{0.061269 J/m^3}{10^{-13} J/m^3}} = 884.73$$

$$R(t_d) = \frac{R(t_0)}{884.730} = \frac{2.01316 \times 10^{26} m}{884.730}$$

$= 2.27545 \times 10^{23} m.$

$= 2.28 \times 10^{23} m$ to 2 decimal places

$$\rho = \rho_0^m \left(\frac{R_0}{R}\right)^3 + \rho_0^r \left(\frac{R_0}{R}\right)^4 \Rightarrow \rho_d = \rho_0^m \left(\frac{R_0}{R_d}\right)^3 + \rho_0^r \left(\frac{R_0}{R_d}\right)^4$$

$$\rho_d = 10^{-9} J/m^3 (884.73)^3 + 10^{-13} J/m^3 (884.73)^4$$

$= 0.75379 J/m^3$

$\rho_d = 0.75 J/m^3$ ~~to~~ 2 decimal places

13