The Open University

Supplementary Material
S357 Space, Time and Cosmology

Course and assignment number:

# Tutor Marked Assignment

**S357 04**

Make sure you know how to complete and send in your TMA and PT3 form: detailed instructions are given in your student handbook (or supplement).

Covering: **Units 13–15**

Cut-off date:
**Friday 26 September 1997**

### Question 1

*This question carries 60% of the marks for this assignment.*

This question concerns a hypothetical universe whose scale factor $R$ evolves in accordance with the Friedmann equations of Section 2.2 of Unit 15:

$$\frac{1}{R}\frac{\mathrm{d}^2R}{\mathrm{d}t^2} = -\frac{1}{2N}[\rho + 3p] \tag{1}$$

$$\frac{1}{R}\frac{\mathrm{d}^2R}{\mathrm{d}t^2} + \left(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 + \frac{kc^2}{R^2} = \frac{1}{2N}[\rho - 3p] \tag{2}$$

where $N = \dfrac{3c^2}{8\pi G} = 1.6077 \times 10^{26}\,\mathrm{kg\,m^{-1}}$.

The energy density $\rho$ consists of two terms:

$$\rho = \rho^{\mathrm{m}} + \rho^{\mathrm{r}}$$

where $\rho^{\mathrm{m}}$ is the energy density of *matter* and $\rho^{\mathrm{r}}$ is the energy density of *radiation*, related to the temperature $T$ by $\rho^{\mathrm{r}} = aT^4$ where $a = 7.5641 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$.

The pressure $p$ is solely due to radiation and is given by $p = \frac{1}{3}\rho^{\mathrm{r}}$.

At time $t_0$, it is found that $H = 2 \times 10^{-18}\,\mathrm{s^{-1}}$, $\rho_0^{\mathrm{m}} = 10^{-9}\,\mathrm{J\,m^{-3}}$, and $\rho_0^{\mathrm{r}} = 10^{-13}\,\mathrm{J\,m^{-3}}$.

(i) Show that

$$\rho = N\left(H^2 + \frac{kc^2}{R^2}\right) \tag{3}$$

$\left(\frac{\rho}{N} - \frac{kc^2}{R^2}\right) = H^2$

(ii) What are the dimensions of $\rho$, $N$, $H^2$ and $R$?

(iii) What is the sign of $k$ in this case? By choosing $k = 1$, find $R(t_0)$. Explain the meaning of $k$ and $R$ in relation to the curvature of space and time, including the case of a 'flat' universe.

(iv) Explain why the energy density $\rho$ and pressure $p$ are given by

$$\rho = \rho_0^{\mathrm{m}}\left[\frac{R_0}{R}\right]^3 + \rho_0^{\mathrm{r}}\left[\frac{R_0}{R}\right]^4 \tag{4a}$$

$$p = \tfrac{1}{3}\rho_0^{\mathrm{r}}\left[\frac{R_0}{R}\right]^4 \tag{4b}$$

at all times. (Account for the powers as well as the constants involved.)

$R_0, R_d, \rho_0$

(v) Suppose decoupling occurs at 3000 K. Find the density of the universe $\rho_{\mathrm{d}}$, and the scale factor $R_{\mathrm{d}}$, at decoupling. (Give your answers to 2 decimal places.)

(vi) Suppose it is known that $H = 2/(3t)$. Find the time of decoupling $t_{\mathrm{d}}$. Use this value of time to find $H(t_{\mathrm{d}})$, and using $R_{\mathrm{d}}$ from part (v) estimate $\rho_{\mathrm{d}}$ by using Equation 3. How does your result for $\rho_{\mathrm{d}}$ compare with the value you obtained in part (v)? Account briefly for any differences.

(vii) Explain very briefly in words what you would need to do in order to find the time at which nuclear reactions occurred throughout the universe. (Do not attempt the algebra.)

### Question 2

*This question carries 40% of the marks for this assignment.*

Write a short essay (about 400 words, say) on the significance of isotropy in cosmology. In particular, you should pay attention to the following points:

(i) What evidence is there to support (or refute) the contention that we live in an isotropic universe?

(ii) In what way does isotropy enter our mathematical models of space and time on the cosmic scale?

(iii) What physical evidence is there that the Universe is not completely isotropic, and what does this tell us about the uniformity of the early Universe?

*General advice on the writing of essays was provided in Unit 8 and should be followed here. However, remember that this essay only carries 40% of the marks for this assignment and that it should be kept short. You might find it helpful to imagine that you are writing a short article for a journal or a magazine, where the editor will demand that every sentence is interesting, accurate and relevant. In this case, about three-quarters of the marks will be given for content, and the rest for organization, balance and coherence.*

$\rho = 1.6077 \times 10^{26} \left(\frac{4}{9t^2}\right.$

$\frac{m^2 s^{-2}}{m^3 kg^{-1} s^{-2}} = kg\, m^{-1}$ $\quad t^2 = \frac{1.6077 \times 10^{26} \times 4}{10^{-13} \quad 9}$