

$$vi) H = \frac{2}{3t}$$

$$\text{but } H = \frac{1}{R} \frac{dR}{dt}$$

$$\therefore \frac{1}{R} \frac{dR}{dt} = \frac{2}{3t}$$

$$dR = \frac{2}{3} \frac{dt}{t}$$

$$\int_{R_d}^{R_0} \frac{dR}{R} = \int_{t_d}^{t_0} \frac{2}{3} \frac{dt}{t}$$

$$[\ln R]_{R_d}^{R_0} = \left[\frac{2}{3} \ln t \right]_{t_d}^{t_0}$$

$$\ln \left(\frac{R_0}{R_d} \right) = \frac{2}{3} \ln \left(\frac{t_0}{t_d} \right) = \ln \left(\left(\frac{t_0}{t_d} \right)^{2/3} \right)$$

$$\therefore \frac{R_0}{R_d} = \left(\frac{t_0}{t_d} \right)^{2/3}$$

$$\text{Rearranging } \left(\frac{R_0}{R_d} \right)^{3/2} = \frac{t_0}{t_d}$$

$$t_d = t_0 / \left(\frac{R_0}{R_d} \right)^{3/2}$$

$$t_d = t_0 \left(\frac{R_d}{R_0} \right)^{3/2}$$

$$\text{With } t_0 = \frac{2}{3H_0} = \frac{2}{3 \times 2 \times 10^{-18} \text{ s}^{-1}} = 3.33333 \times 10^{17} \text{ s}$$

$$\text{and } \frac{R_0}{R_d} = 884.73$$

$$t_d = 3.33333 \times 10^{17} \text{ s} (884.73)^{-3/2} = 1.26667 \times 10^{13} \text{ s}$$

$$H(t_d) = \frac{2}{3t} = \frac{2}{3 \times 1.26667 \times 10^{13} \text{ s}} = 5.26 \times 10^{-14} \text{ s}^{-1}$$