$R_d = 2.27545 \times 10^{23} m, k=1$

$$\rho = N\left(H^2 + \frac{kc^2}{R^2}\right) \Rightarrow \rho_d = N\left(H_d^2 + \frac{kc^2}{R_d^2}\right)$$

$$\rho_d = 1.6077 \times 10^{26} kgm^{-1}\left((5.26314 \times 10^{-14} s^{-1})^2 + \frac{1 \times (3 \times 10^8 m/s)^2}{(2.27545 \times 10^{23} m)^2}\right)$$

$$= 1.6077 \times 10^{26} kgm^{-1}\left(2.77006 \times 10^{-27} s^{-2} + 1.73823 \times 10^{-30} s^{-2}\right)$$

$$= 1.6077 \times 10^{26} kgm^{-1} \times 2.77718 \times 10^{-27} s^{-2}$$

$$= 0.44562 J/m^3$$

(I have used $k=1$ (closed universe) with $H = \frac{2}{3t}$ (open universe), which is an inconsistency.

With $k=0$ and $H = \frac{2}{3t}$ and $z = \frac{R(t_0)}{R(t_d)} - 1$ for the cosmic background radiation

$$z = \frac{R(t_0)}{R(t_d)} - 1 = \frac{T_d}{T_0} = \frac{3000}{2.73} = 1098.9$$

$$\frac{R(t_0)}{R(t_d)} - 1 = 1098.9 \Rightarrow \frac{R(t_0)}{R(t_d)} = 1099.9$$

sub $\frac{R(t_0)}{R(t_d)} = 1099.9$ into $t_d = t_0\left(\frac{R(t_0)}{R(t_d)}\right)^{-3/2}$

$$\therefore t_d = 3.33333 \times 10^{17} s (1099.9)^{-3/2}$$

$$= 9.13794 \times 10^{12} s \quad (= 288,263 \text{ yrs}) *$$

$$H(t_d) = \frac{2}{3t_d} = \frac{2}{3 \times 9.13794 \times 10^{12} s} = 7.29559 \times 10^{-14} s^{-1}$$

$$\rho = N\left(H^2 + \frac{kc^2}{R^2}\right) \Rightarrow \rho_d = N\left(H_d^2 + \frac{kc^2}{R_d^2}\right)$$

$$= 1.6077 \times 10^{26} kgm^{-1}\left((7.29559 \times 10^{-14} s^{-1})^2 + \frac{0 \times c^2}{R^2}\right)$$

$$= 0.85571 J/m^3$$

which is greater than the answer in (v)

* Book 4 gives $t_d \simeq 300,000$ yrs but they used $z \simeq 1000$ and $t_d = \frac{10^{10} \text{yrs}}{(1000)^{3/2}} \simeq 3 \times 10^5$ yrs