TMA 04 S357

1) i) $\frac{1}{R}\frac{d^2R}{dt^2} = -\frac{1}{2N}(\rho + 3p)$ ①

$\frac{1}{R}\frac{d^2R}{dt^2} + \left(\frac{1}{R}\frac{dR}{dt}\right)^2 + \frac{kc^2}{R^2} = \frac{1}{2N}(\rho - 3p)$ ②

② − ①

$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 + \frac{kc^2}{R^2} = \frac{1}{2N}(\rho - 3p) - -\frac{1}{2N}(\rho + 3p)$

$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 + \frac{kc^2}{R^2} = \frac{\rho}{N}$

$\frac{1}{R}\frac{dR}{dt} = H$ so $\left(\frac{1}{R}\frac{dR}{dt}\right)^2 = H^2$

$\frac{\rho}{N} = H^2 + \frac{kc^2}{R^2}$

$\rho = N\left(H^2 + \frac{kc^2}{R^2}\right)$ 4.

ii) Energy density phase dimension $\frac{E}{V} = \frac{J}{m^3} = J\,m^{-3}$

$N = \frac{3c^2}{8\pi G}$, so dimensions are $\frac{m^2 s^{-2}}{m^3 kg^{-1} s^{-2}} = kg\,m^{-1}$

$H^2 = \left(\frac{1}{R}\frac{dR}{dt}\right)^2$ so dimensions are $\left(\frac{m s^{-1}}{m}\right)^2 = s^{-2}$

R is measure of length so dimension are m. 4.

iii) $\rho = N\left(H^2 + \frac{kc^2}{R^2}\right)$ ③

$\rho = \rho^m + \rho^r$

$= (1 \times 10^{-9} + 1 \times 10^{-13})\,J/m^3$

$= 1.0001 \times 10^{-9}\,J/m^3$

Rearrange 3 to make k the subject

$\frac{\rho}{N} = H^2 + \frac{kc^2}{R^2}$

$k = \frac{R^2}{c^2}\left(\frac{\rho}{N} - H^2\right)$