

Also with  $\frac{R(t_0)}{R(t_d)} = 1100$

$$\begin{aligned} \rho_d &= \rho_0^m \left( \frac{R_0}{R_d} \right)^3 + \rho_0^r \left( \frac{R_0}{R_d} \right)^4 \\ &= 10^{-9} \text{ J/m}^3 (1100)^3 + 10^{-13} \text{ J/m}^3 (1100)^4 \\ &= (1.331 + 0.1464) \text{ J/m}^3 \\ &= 1.47741 \text{ J/m}^3 \end{aligned}$$

With  $k=0$  in Eq (3) ( $k$  then becomes irrelevant)  
with  $H(t_d) = 5.26314 \times 10^{-14} \text{ s}^{-1}$

$$\rho = N \left( H^2 + \frac{k c^2}{R^2} \right) \quad \therefore \quad \rho_d = N \left( H_d^2 + \frac{k c^2}{R_d^2} \right)$$

$$\begin{aligned} \rho_d &= 1.6077 \times 10^{-26} \text{ kg m}^{-3} \left( (5.26314 \times 10^{-14})^2 + \frac{0 \times c^2}{R_d^2} \right) \\ &= 0.44534 \text{ J/m}^3 \end{aligned}$$

Rest of this part answered overleaf.

(I have gone to all this effort because I think it is unclear what the question wants.

If the universe is closed, then it will have expanded less since decoupling than for a critical universe, so for given values of  $\rho_0^r$  and  $H_0$ ,  $\rho_d$  for a closed universe will be less than for a critical universe, <sup>using the methods I have shown.</sup>

In parts (iii) and (v) we calculated  $R(t_0)$ ,  $R(t_d)$  and  $\frac{R(t_0)}{R(t_d)}$  for a closed universe, and in part

(vi) using  $H = \frac{2}{3t}$  (and finding that  $\frac{R(t_0)}{R(t_d)} = \left( \frac{t_0}{t_d} \right)^{3/2}$ )

we found  $t_d$ , but  $\frac{R(t_0)}{R(t_d)} > 884.73$  for a critical universe, since decoupling is less between  $t_0$  and  $t_d$ .

As if we use  $\frac{R(t_0)}{R(t_d)} = z+1$ , where  $z$  is the redshift

it experienced by the background radiation