Energy density of radiation

$$\rho^r \propto \frac{1}{\text{Volume}} \times \frac{1}{\text{wavelength}}$$

$$\propto \frac{1}{R^3} \times \frac{1}{R} = \frac{1}{R^4} \Rightarrow \rho^r R^4 = \text{constant}$$

radiation density ρ_1^r at any t_1 related to radiation density ρ_0^r at present time t_0 by

$$\frac{\rho_1^r (R(t_1))^4}{\rho_0^r (R(t_0))^4} = \frac{\text{constant}}{\text{constant}} = 1 \Rightarrow \rho_1^r = \rho_0^r \left(\frac{R(t_0)}{R(t_1)}\right)^4$$

Matter density, $\rho^m \propto \frac{1}{R^3} \Rightarrow \rho^m R^3 = \text{constant}.$

$\therefore$ matter density ρ_1^m at any time t_1 related to matter density at present time ρ_0^m by

$$\frac{\rho_1^m (R(t_1))^3}{\rho_0^m (R(t_0))^3} = \frac{\text{constant}}{\text{constant}} = 1$$

$$\therefore \rho_1^m = \rho_0^m \left(\frac{R(t_0)}{R(t_1)}\right)^3$$

$$\rho = \rho^m + \rho^r$$

$$\therefore \rho_1 = \rho_1^m + \rho_1^r$$

$$\rho_1 = \rho_0^m \left(\frac{R(t_0)}{R(t_1)}\right)^3 + \rho_0^r \left(\frac{R(t_0)}{R(t_1)}\right)^3$$

or $\rho = \rho_0^m \left(\frac{R_0}{R}\right)^3 + \rho_0^r \left(\frac{R_0}{R}\right)^4$ equivalently where ρ, R are at time t, ρ_0, R_0 at time t_0

It is given in the question that pressure, $P = \frac{\rho^r}{3}$

So if $\rho^r = \rho_0^r \left(\frac{R_0}{R}\right)^4$

$$\therefore P = \frac{1}{3}\rho^r = \frac{1}{3} \times \rho_0^r \left(\frac{R_0}{R}\right)^4$$

$P = \frac{1}{3}\rho_0^r \left(\frac{R_0}{R}\right)^4$ as required.

12