Course and assignment number:

Tutor Marked Assignment

S357 03

Make sure you know how to complete and send in your TMA and PT3 form: detailed instructions are given in your student handbook (or supplement).

Covering: **Units 9–12**

Cut-off date:
Friday 22 August 1997

This assignment consists of two questions respectively weighted at 50% and 50% of the total marks for the assignment.

Question 1

This question carries 50% of the marks for this assignment and relates mainly to Units 9 and 10&11. (You will also need to refer to Unit 2.)

Write an essay explaining how ideas about gravity have developed from the time of Newton to the present day. Your account should include:

(i) A precise statement of Newton's law of gravity, together with a description of the motion of planets and of light according to this law and Newton's laws of motion;

(ii) An explanation of how general relativity dispenses with the notion of a gravitational force, replacing it with some other concept;

(iii) A description of the effect of gravity on the motion of material bodies and of light, and on the energy of photons, according to general relativity;

(iv) An explanation of why *both* Newton's theory of gravity and general relativity are in current use.

You should mention experimental evidence that is relevant to the points you are making, but this should not be a dominant part of your essay.

Note that this question requires you to combine relevant material from several units, in a coherent manner, as is frequently the case with questions set in Part III of the final examination. General advice on essay writing was given in Unit 8, but you might find it useful to keep the following points in mind.

- Full marks may be obtained for a well-organized and clearly expressed essay of about 600–1000 words.
- As guidance for the content of the essay, topics of principal importance are listed.
- Approximately 75% of the marks for this question are reserved for the coverage of these principal points.
- The remaining 25% of the marks for this question are for your explanation of supporting points, and for the overall clarity, balance and organization of the essay, including your overall conclusions in a final paragraph.

Question 2

This question carries 50% of the marks for this assignment.

The question concerns motion in Schwarzschild spacetime.

A test particle moves about a static, spherically symmetric, body of mass M in the plane $\theta = \pi/2$, where θ is one of the standard coordinates (t, r, θ, ϕ) of the Schwarzschild metric.

(i) Let the particle's world-line be parameterized by the proper time τ elapsed along it. Show that for this particle

$$(\Delta\tau)^2 = \left(1 - \frac{2MG}{c^2 r(\tau)}\right)(\Delta t)^2 - \frac{r^2(\tau)}{c^2}(\Delta\phi)^2 - \frac{1}{c^2\left(1 - \dfrac{2MG}{c^2 r(\tau)}\right)}(\Delta r)^2. \qquad (1)$$

(ii) If the speed is small compared to c, and if $MG/rc^2 \ll 1$, we may take the approximations $\Delta\phi \approx (v_\phi/r)\,\Delta t$ and $\Delta r \approx v_r\,\Delta t$, where v_ϕ and v_r are the components of velocity in the ϕ and r directions, so that the speed is $v \approx \sqrt{v_\phi^2 + v_r^2}$. Show, under these assumptions, that

$$\Delta\tau \approx \Delta t\left(1 - \frac{MG}{rc^2} - \frac{v^2}{2c^2}\right). \qquad (2)$$

[Hint: When $|x| \ll 1$, $\sqrt{1-x} \simeq 1 - x/2$ and $1/(1-x) \approx 1 + x$.]

(iii) One of the geodesic equations of Schwarzschild spacetime may be written in the form:

$$\left(1 - \frac{2MG}{c^2 r(\tau)}\right)\frac{\mathrm{d}t(\tau)}{\mathrm{d}\tau} = N \qquad (3)$$

where N is a constant. (You are *not* required to derive this equation.)

Combine Equations 2 and 3 to show, when both v^2 and MG/r are *small* compared to c^2, that a test particle of mass m which moves in the spacetime curved by a static, spherically symmetric, body of mass M is subject to the condition

$$\frac{1}{2}mv^2 - \frac{mMG}{r} \approx \text{constant}. \qquad (4)$$

(iv) Give a physical interpretation of Equation 4 in Newtonian mechanics.