Model Answers

These model answers break the solution of each problem down into five sequential steps. These steps are always the same, and when they are taken one at a time, the apparent complexity of the problem can be reduced considerably.

For questions which you answered correctly, checking your method against these five steps can help you to get a feel for this simple but robust problem-solving technique.

For questions which you did not answer correctly, checking your chain of reasoning against each of the five steps in turn can help you to identify exactly where you went wrong.

General form of model answer:

Q00

Objects given: The problem is translated and analysed into the mathematical/physical objects which are given.
To be found: The unknown to be found is defined in terms of the mathematical/physical objects given.
Direct solution: A solution is applied which – in one step if possible – links the given mathematical/physical objects directly to the unknown to be found.
Result: The result is computed and the appropriate item(s) are chosen.
Corroboration: An alternative method of solution is applied in order to check and corroborate the result.

Model answers:

Q1 D

Objects given: Two displacement vectors of equal magnitude.
To be found: The angle between them.
Direct solution: Write down the equation for the cosine of the angle, and plug in the values given.
Result: The magnitude of both vectors is $\sqrt{8}$ and their scalar product is equal to 4, so the cosine of the angle between them is $4/8 = 1/2$, and the angle itself therefore is 60°: item (D).
Corroboration: Sketch the vectors.

Q2 F

Objects given: A particle with mass m, position $\boldsymbol{x}$, velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ at time t.
To be found: The single expression given in these variables, which does not change when the time scale is recalibrated.
Direct solution: If all units of time in just one expression cancel out, that will be the right expression.
Result: Item (F).
Corroboration: With $t' = 1000t$, then $v' = v/1000$ and $a' = a/1000000$. So item (F) alone remains invariant.

Q3 C

Objects given: A particle with mass m, position $\boldsymbol{x}$, velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ at time t.
To be found: The single expression given in these variables, which does not change when the length scale is recalibrated.
Direct solution: If all units of length in just one expression cancel out, that will be the right expression.
Result: Item (C).
Corroboration: With $x' = x/1000$, then $v' = v/1000$ and $a' = a/1000$. So item (C) alone remains invariant.

Q4 E

Objects given: A particle with mass m, position $\boldsymbol{x}$, velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ at time t.
To be found: The single expression given in these variables, which does not change under a translation of the spatial co-ordinate origin.
Direct solution: If just one expression does not involve $\boldsymbol{x}$ at all, that will be the right expression.
Result: Item (E).
Corroboration: Under $\boldsymbol{x}' = \boldsymbol{x} - \Delta$, $\boldsymbol{v}$ and $\boldsymbol{a}$ are invariant. So item (E) alone remains invariant.