S357 Space, Time and Cosmology

S357, CMA 43 Answers (1997)

Q1 $\boxed{\text{A}}$ From the given coordinate functions it can be seen that $(0, 2\tau_0)$ corresponds to $\tau = 0$, and $(28, 2\tau - 6)$ corresponds to $\tau = 2$. It then follows from the $\mathscr{L}_{\mathscr{C}}(\mathrm{P}, \mathrm{Q})$ given in Section 4 of Unit 10 & 11 that the required length is given by

$$\int_{\tau_P}^{\tau_Q} \left[\left(\dot{x}^1(\tau)\right)^2 + \left(\dot{x}^2(\tau)\right)^2 \right]^{1/2} \mathrm{d}\tau = \int_0^2 \left[(14\tau)^2 + (-3)^2 \right]^{1/2} \mathrm{d}\tau$$
$$= \int_0^2 \left(196\tau^2 + 9\right)^{1/2} \mathrm{d}\tau$$

Q2 $\boxed{\text{D}}$ The correct expression for the metric of a plane in radial polar coordinates is

$$\Delta\ell = \left[(\Delta\rho)^2 + \rho^2 (\Delta\phi)^2 \right]^{1/2}.$$

All the other statements are supported by Sections 5 to 7 of Unit 10 & 11. Note that in the case of option A, we assume that the plane is non-rotating.

Q3 $\boxed{\text{B,D}}$ In spacetime, geodesics are the paths of particles that are not subject to any non-gravitational forces. There are the pathways of maximum proper time. In a plane, straight lines are geodesics and there are infinitely many through any point. On the surface of a sphere great circles are geodesics and there are, again, infintely many through each point; though only one great circle passes through two given points.

Q4 $\boxed{\text{D,F}}$ The geodesic equations provide differential equations that may be used to determine q^1 and q^2 at points on a geodesic as functions of a parameter s that varies along the geodesic. (By eliminating s the relationship between q^1 and q^2 that defines the geodesic may then be determined.) It follows that L must be (implicitly) a function of s and g_1 and g_2 must be known functions of q^1 and q^2. In fact, for the metric of equation 33, g_1 must be a constant and g_2 a function of q^1.

Q5 $\boxed{\text{D}}$

A is false: only the Ricci curvature necessarily vanishes in the absence of sources.
B is false: the metric coefficients depend also on the choice of coordinate system.
C is false: the components of the Ricci curvature are combinations of the components of the Riemann curvature.
D is true: this follows immediately from the field equations.
E is false since D is true.

Q6 $\boxed{\text{G}}$ Using equation 68b of Unit 10 & 11 with $g_1 = 1$ and $g_2 = \cos^2(\lambda q^1)$ gives

$$\mathscr{K}(q^1, q^2) = \frac{1}{4\cos^2(\lambda q^1)} \left(-2\lambda \cos(\lambda q^1) \sin(\lambda q^1)\right)^2 - \frac{1}{2\cos^2(\lambda q^1)} \left(-2\lambda^2 \cos^2(\lambda q^1) + 2\lambda^2 \sin^2(\lambda q^1)\right)$$
$$= \lambda^2 \sin^2(\lambda q^1) - \lambda^2 \tan^2(\lambda q^1) + \lambda^2$$

As $q^1 = 0, q^2 = 0$ this gives $\mathscr{K}(0,0) = \lambda^2$.

Q7 $\boxed{\text{C}}$ Since $\mathscr{K}(0,0) > 0$ the small circle will be in a region of positive curvature, where C will be less than $2\pi r$.

Q8 $\boxed{\text{A}}$ Due to the positive curvature in the neighbourhood of (0,0) the sum of the angles must exceed π.

Q9 $\boxed{\text{G}}$ The surface must have negative curvature *everywhere*. None of the surfaces described has this property. (A torus has negative curvature at some points, but not everywhere.)