b) $\lambda_n = \frac{(2m+1)^2 \pi^2 D}{L^2}$ (14)

$f_n = \sin\left(\frac{(2m-1)\pi z}{L}\right)$

c) $B_n = \frac{4 T_0}{\pi(2m-1)}$

d) $m=1$ $\quad T_f = \frac{4T_0}{\pi} \sin\left(\frac{\pi z}{L}\right) e^{-\frac{\pi^2 D t}{L^2}}$

e) $\cancel{T_f = \sum e^{-\frac{\pi^2}{L^2} D t} = \frac{4}{\pi} \int \sin \frac{\pi z}{L} e^{-\pi^2 \frac{D t_f}{L^2}}}$

at $t = t_f$ $\quad T_f = \frac{4T_0}{\pi} \sin\left(\frac{\pi z}{L}\right) e^{-\pi^2 \frac{D t_f}{L^2}}$

the heat is redistributed

$\therefore T_f = \frac{4T_0}{\pi} \int \sin \frac{\pi z}{L} e^{-\pi^2 \frac{D t_f}{L^2}} dz$

$= \frac{4T_0}{\pi} \left[-\frac{L}{\pi} \cos \frac{\pi z}{L} \right]_0^L e^{-\pi^2 \frac{D t_f}{L^2}}$

$= \frac{4T_0}{\pi} \left[-\frac{L}{\pi} \left(\cos \pi - 1 \right) \right] e^{-\pi^2 \frac{D t_f}{L^2}}$

$= \frac{8T_0}{\pi^2} e^{-\pi^2 \frac{D t_f}{L^2}}$