$$\therefore x_n = A\alpha^n + \frac{1}{2(1-\alpha)}$$

$$n=0 \quad 0 = x_0 = A + \frac{1}{2(1-\alpha)}$$

$$\therefore A = \frac{-1}{2(1-\alpha)}$$

$$\therefore x_n = \frac{-\alpha^n}{2(1-\alpha)} + \frac{1}{2(1-\alpha)}$$

$$= \frac{1-\alpha^n}{2(1-\alpha)}$$

d) $$\langle x_n^2 \rangle = \alpha^2 \langle x_n^2 \rangle + \frac{2\alpha\langle x_n \rangle}{2} + \frac{1}{3}$$

$$\langle x_n^2 \rangle (1-\alpha^2) = \alpha \langle x_n \rangle + \frac{1}{3}$$

$$\langle x_n^2 \rangle = \frac{3\alpha\langle x_n \rangle + 1}{3(1-\alpha^2)}$$

$$= \frac{3\alpha\left(\frac{1-\alpha^n}{2(1-\alpha)}\right) + 1}{3(1-\alpha^2)}$$

$$V = \langle x_n^2 \rangle - \langle x_n \rangle^2$$

$$= \frac{3\alpha(1-\alpha^n) + 2(1-\alpha)}{6(1-\alpha^2)(1-\alpha)} - \frac{(1-\alpha^n)^2}{4(1-\alpha)^2}$$