4) $\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial z^2}$

(12)

$\cancel{T = T_0 \text{ when } t = 0, \ z \ 0 < z < L}$

$\cancel{T =}$

$T(z, t) = T_0$

$T(0, t) = 0 \ , \ T(L, t) = 0$

$T' Z = D T Z''$

$\frac{T'}{T} = \frac{D Z''}{Z} = C$

$T' = CT \Rightarrow T = Ae^{Ct}$

$\cancel{Z'' - CZ = 0 \Rightarrow}$

$C < 0$ else $T \to \infty$ as $t \to \infty$

$\therefore T = Ae^{-k^2 t}$

$\therefore \frac{DZ''}{Z} = -k^2 \Rightarrow Z'' + \frac{k^2}{D} Z = 0$

$Z = B \sin\left(\frac{k}{\sqrt{D}} z\right) + D \cos\left(\frac{k}{\sqrt{D}} z\right)$

$T = \left(B \sin \frac{k}{\sqrt{D}} z + D \cos\left(\frac{k}{\sqrt{D}} z\right)\right) e^{-k^2 t}$

$T(0, t) = 0 \Rightarrow 0 = (0 + D) e^{-k^2 t}$

$\therefore D = 0$