$$T(L,t) = 0$$

$$\Rightarrow 0 = \left(B \sin \frac{k}{\sqrt{D}} L\right) e^{-k^2 t}$$

$$\Rightarrow \frac{k}{\sqrt{D}} L = n\pi$$

$$\Rightarrow k = \frac{n\pi\sqrt{D}}{L}$$

$$\therefore T = \sum B \sin \frac{\pi n}{L} z \, e^{-\frac{n^2\pi^2 D}{L^2} t}$$

b) ~~$\lambda_n = \frac{n^2\pi^2 D}{L^2}$, $f_n = B_n$~~

$$T_0 = \sum B_n \sin \frac{n\pi z}{L}$$

$$B_n = \frac{2}{L}\int_0^L T_0 \sin \frac{n\pi z}{L}$$

$$= \frac{2T_0}{L}\left[-\frac{L}{\pi n} \cos \frac{n\pi z}{L}\right]_0^L$$

$$= \frac{2T_0}{L}\left[\frac{-L}{\pi n} \cos n\pi + \frac{L}{n\pi}\right]$$

$$= \frac{2T_0}{\pi n}\left(1 - (-1)^n\right) = \frac{4}{\pi n}\begin{cases} n \text{ odd} \\ 0 \; n \text{ even}\end{cases}$$

$$\therefore B_m$$

$$T_0 = \sum \frac{4T_0}{\pi(2m-1)} \sin\left(\frac{(2m-1)\pi z}{L}\right) e^{-\frac{(2m-1)^2\pi^2 D}{L^2} t}$$