31

Players take it in turn to choose a number between 1 and 6. A running total is kept. The player who makes the running total 31 wins.

After playing the game, we discovered that 24 was a crucial number in this game. If you say 24, your opponent loses. Working backwards from this we found that the numbers 3, 10, and 17 control the game in a similar way.

These numbers were always minus 7 from the number before, as 6+1=7; (Max no.) + (Min no.) = 7.

You can always get the game to these numbers if you say 3 first, because whatever they say you can make it up to 10, 17 etc.

From this we created a formula.
Let the target number (31 in this case) = **m**
Limits = **a** and **b**
Start number = **S**

Equation of starting number: **S = m - ((a+b).INT[m/(a+b)])**

If **S** = 0, or a number you cannot play (say **b+2**) let your opponent begin.

Steps are simply **a+b**.

Variations

As our formula allows for any target number (**m**) and set the limits at **a** and **b**, rather than **1** and **b**, the game can be played with a set of numbers between any 2 points with the same strategy; *for example:*

Players may choose any number between 3 and 7, and the player who makes the running total 43 wins.

Let m = 43, a= 3 and b= 7

S = 43 – ((3+7).INT[m/(a+b)])
S = 43 – (10*INT[43/10])

S = 3

We play first, and begin on 3. After our opponent plays we make our turn bring the running total to the next of our target points (in steps of 10); 3, 13, 23, 33, 43.

Our strategy could also be used in a game with a uniform limited use of all the numbers a to b (assuming you can still reach you target number without running out of numbers); as effectively you pair the numbers up, and your opponent would run out of the opposite of the pair at the same time as you ran out of the other half of the pair.

The only problem our strategy would face in this variation of the game would be if your opponent used the full quota of the number opposite in pairing to your starting number. Unless your opponent has sussed the strategy and has realised this, this is a situation you are unlikely to fa ce, and therefore you should still win far more often than lose. (Maybe play best of 3 if your opponent suggests limited number use)

Piles

There are two piles of matches. Players can either take any number they like from one pile, or the same number from both piles. The winner is the player who takes the last match.

In the game description we are not given a specified number of matches in each pile, neither is there a limit on the size of either pile. Therefore we must find a tactic that will work for any start point.

You can win immediately if your opponent leaves you with 2 even piles, or leaves just one pile, but unfortunately a smart opponent will not give you this opportunity unless you force them into a position such that you will win, no matter what move they make.

Because there is no specified starting number of matches, it makes sense to work out a tactic from the end of the game. The first losing position is 2 – 1; as however your opponent plays you can win.

Your opponent can only leave you with:

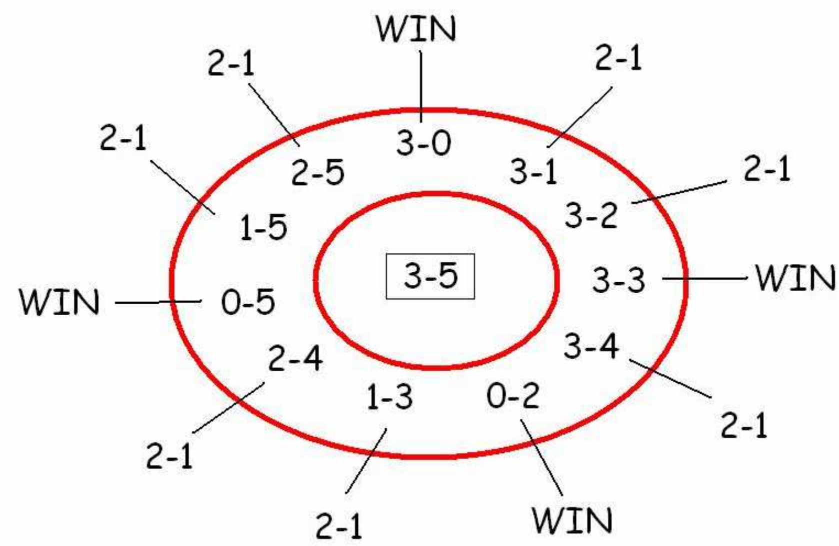
- 1-1; Take both to win
- 2-0; Take all to win
- 0-1; Take the last match to win

Now you could create a new object of the game; to force your opponent to face 2 – 1.

The next logical step forward in finding a tactic to win would be to attempt to find a position where, whatever the opponent takes we can win, or leave them facing 2 – 1. If such a position exists, we must apply some constraints:

- The match piles cannot be 1 apart because the opponent could force us to face 2 – 1, and therefore cause us to lose.
- Neither pile can contain a 2 or a 1 because the opponent can force us to face 2 - 1.

If we try 3, the lowest number not yet used, and 5, because it is two higher than 3, we can construct a diagram showing every possible outcome from this point.

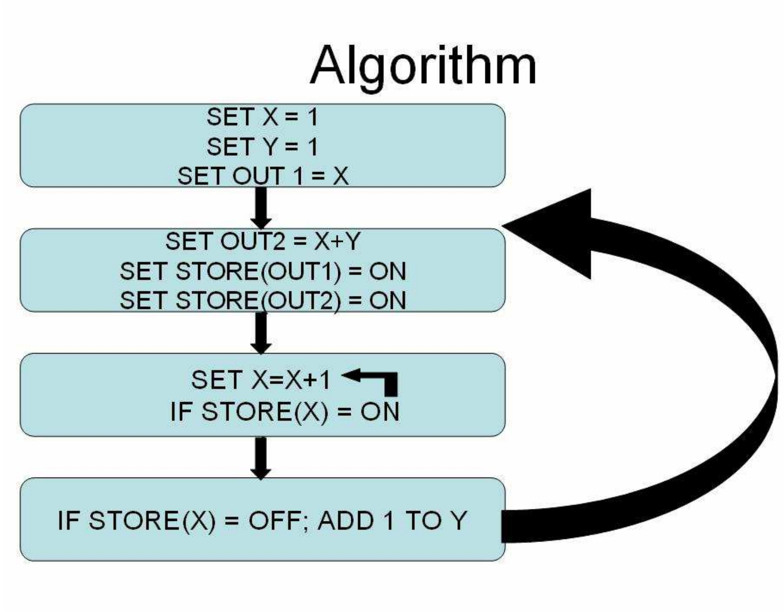


After looking at all the possible outcomes we can say that 5 -3 is the next losing position. If we assume that more losing positions can be calculated in the same way, we can construct a list of losing positions.

For example:
The next losing position must not contain 1, 2, 3 or 5 and must have a difference of three between the numbers. Therefore the next losing position must be 7-4, as 4 is the lowest available number and 7 is three higher than 4.

If we continue in this fashion we can construct a list of losing positions: 1-2, 3-5, 4-7, 6-10, 8-13, 9-15 and so on.

We created an algorithm to generate losing positions for us, using the same method to calculate them.



Where SET X indicates the lower number in the losing position and SET Y indicates the numerical gap in between the two numbers. OUT 1 is the lower number and OUT 2 calculates the higher number. This algorithm can continue to calculate losing positions forever.

Although this method is valid, and creates the right pairs, we thought that there must be a still more efficient way to calculate these losing positions. After some research we discovered that the losing combinations create two sequences, the Upper and Lower Wythoff Sequences; and that there was an equation for both of these sequences.

Lower Wythoff Sequence: 1, 3, 4, 6, 8, 9, 11, 12, 14, 16, 17, 19…

$$\mathrm{INT}\left(\frac{(\sqrt{5}+1)}{2} * n\right)$$

Upper Wythoff Sequence: 2, 5, 7, 10, 13, 15, 18, 20, 23, 26, 28, 31…

$$\mathrm{INT}\left(\left(\frac{(\sqrt{5}+1)}{2}\right)^2 * n\right)$$

If you pair these sequences, you receive the "Losing combinations" list. This enables us to conclude that the tactic of forcing the opponent to face a losing position each go is a solid one, as it is based on a mathematical pattern.

Having concluded this, a player who knows this tactic will always wish to go first as they can immediately take an appropriate number of matches from one or both piles to leave a losing position for their opponent, unless the game starts with a losing combination of matches in which case they will go second. From then on the player will force their opponent into a losing position every go until they eventually win.

Without changing the fictional context of the game (piles of something) we have been unable to think of a variation of this game in which you could apply the same strategy.

You could play the game with more than one opponent but you would not be able to take control of the game, because you can't set the game up for yourself if there are two people taking turns before you.

You could play with more than two piles of matches. In this situation you could immediately eliminate the extra piles and then apply the same tactics as you would for this and essentially becomes the same game.

You could play so that you lose (as opposed to win) if you take the last match, although this would not use the same tactics as you would have to force your opponent down to one match left to win.

Cartesian Dash

A rectangular grid is used. The first player puts a cross in the bottom left hand square. A player must place a cross in the square immediately right, diagonally up and right, or directly above the previously marked square. The winner is the person who puts the cross in the top right hand grid.

There are 6 types of grid you can create for this game. An odd by odd square, an even by even square, an odd by even rectangle, an even by odd rectangle, an odd by odd rectangle and an even and even rectangle . We have deduced strategies for each of these games that work for any number of rows or columns.

Odd Squares

The most basic square in this category is a 3*3 square:

X	X	
X	X	

To win you would want to go **first**, giving your opponent the red Xs as possible moves.

X	X	X
X		
X		

		X
	X	
X		

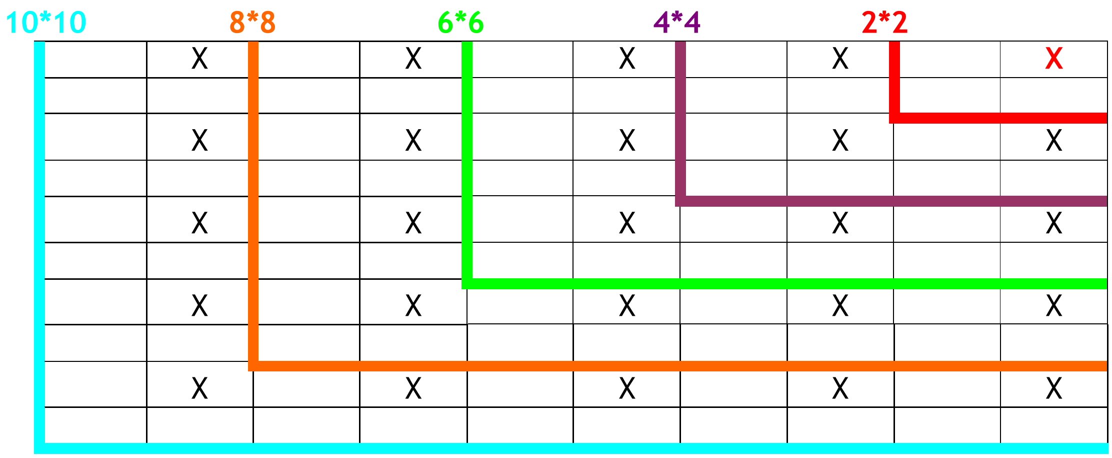
		X
		X
X	X	X

As you can see from the above grids, you can win no matter how the second player places their move.
We can extend this to show "power squares", wherever your opponent plays on each turn you can place your X on a power square and eventually win the game.

11*11		9*9		7*7		5*5		3*3		
X		X		X		X		X		X
X		X		X		X		X		X
X		X		X		X		X		X
X		X		X		X		X		X
X		X		X		X		X		X
X		X		X		X		X		X

Even Squares

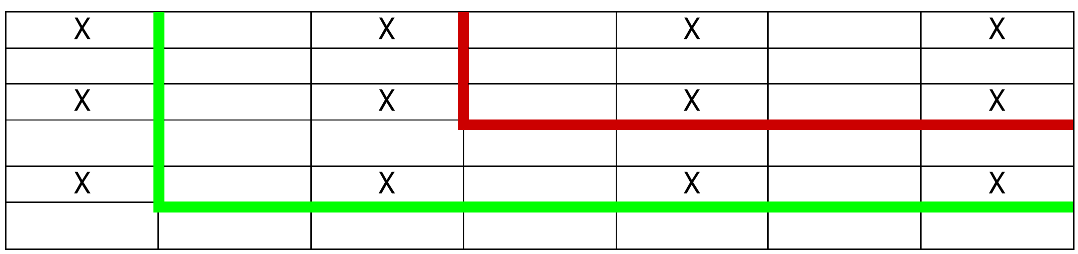
When faced with an even square, always go second; as essentially the bottom left hand corner is not a power square anymore, so your first move will place you in a power square.



Even by Odd Rectangle

(A rectangle with an even number of columns and odd number of rows)

Go second, as after your opponent plays in the bottom left hand corner you can gain control by playing the power square to the right.



Odd by Even Rectangle

(A rectangle with an odd number of columns and even number of rows)

Again, go second, as after your opponent plays in the bottom left hand corner you can gain control by playing directly above.

*Odd by Odd Rectangle**

Go first, as the bottom left corner of any odd by odd rectangle is a power square (same strategy as an odd square).

X		X		X		X
X		X		X		X
X		X		X		X
X		X		X		X

*Even by Even Rectangle**

Go second, as you will be able to use your first move to play on a power square (same strategy as an even square)

	X		X		X		X
	X		X		X		X
	X		X		X		X

**We realised that the strategy for an odd square was the same as an odd by odd rectangle as it is a subset of the category; and similarly for even squares and even by even rectangles.*

Summary of strategy:

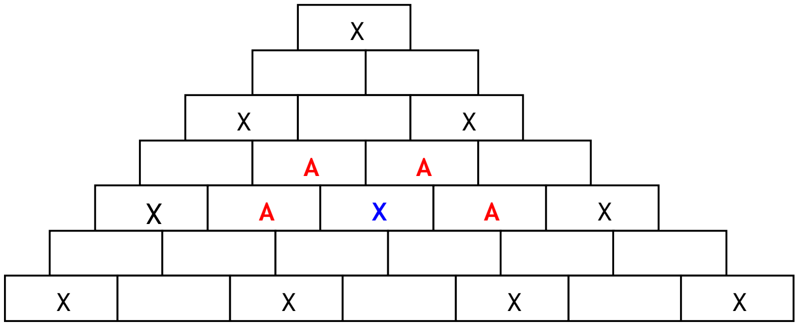
- Go first if you are playing on a grid with both sides odd.
- Go second otherwise.
- ONLY play on power squares.

Adaptation of the Cartesian Square

<u>*Pyramid style:*</u>

- Play begins anywhere on the bottom row
- You can move into any adjacent square not on the row below.

For example, if the blue **X** were played, the next player could mark any of the blocks marked with a red **A**.



Strategy:

- If there is an odd number of rows go first.
- If there is an even number of rows go second.
- Play only on Power blocks.