

$$\begin{aligned}
 2) a) P(\text{correct answer}) &= \phi \\
 &= P(\text{knows correct answer}) \\
 &\quad + P(\text{Does not know correct answer but picks correctly}) \\
 &= \theta + (1 - \theta) \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2}\theta = \frac{\theta + 1}{2} = \phi
 \end{aligned}$$

$$b) \hat{\phi} = \frac{1}{2} + \frac{\hat{\theta}}{2}$$

$\hat{\theta}$ is Bernoulli.

y correct answers

$$P(y) = \binom{100}{y} \phi^y \times (1 - \phi)^{100-y}$$

$$\begin{aligned}
 \frac{dP(y)}{d\phi} &= \binom{100}{y} (y \phi^{y-1}) (1 - \phi)^{100-y} + \binom{100}{y} \phi^y (1 - \phi)^{99-y} (-1) \\
 0 &= y(1 - \phi) - \phi(100 - y) \quad \left[\text{Dividing by } \binom{100}{y} \phi^{y-1} (1 - \phi)^{99-y} \right]
 \end{aligned}$$

$$100\phi - \phi y = y - \phi y$$

$$\hat{\phi} = \frac{y}{100}$$

$$c) \frac{\hat{\theta}}{2} = \frac{y}{100} - \frac{1}{2} = \frac{y - 50}{100} \Rightarrow \hat{\theta} = \frac{y - 50}{50}$$

$$\left[\text{since } \phi = \frac{1}{2} + \frac{\hat{\theta}}{2} \Rightarrow \hat{\theta} = \hat{\phi} - \frac{1}{2} \right]$$