(5) $\therefore$ C. $P(1)$:

$\cancel{\ }$

$1^2 - 2^2 + 3^2 = (1+1)(2\times1+1)$

$6 = 6$

$P(n)$ is $1^2 - 2^2 \ldots + (2n+1)^2 = (n+1)(2n+1)$

$1^2 - 2^2 + \ldots + (2n+1)^2 - (2n+2)^2 + (2n+3)^2$

$= (n+1)(2n+1) - (2n+2)^2 + (2n+3)^2$

$= (n+1)(2n+1) - \cancel{4n^2} - 8n - 4 + \cancel{4n^2} + 12n + 9$

$= (n+1)(2n+1) + 4n + 5$

$= 2n^2 + 3n + 1 + 4n + 5 = 2n^2 + 7n + 6$

$= (2n+3)(n+2) = (n+2)(2n+3)$

$= (n+1+1)(2(n+1)+1)$

$\therefore P(n+1)$ true

m odd $1 - 2^2 + 3^2 \ldots + m^2$

$m = 2n+1 \Rightarrow n = \frac{m-1}{2}$

$\therefore 1 - 2^2 + 3^2 + \ldots + m^2 = \left(\frac{m-1}{2} + 1\right) m = \left(\frac{m+1}{2}\right) m$

m even $1 - 2^2 + 3^2 + (m-1)^2 - m^2$

$= \left(\frac{m-1+1}{2}\right)(m-1) - m^2$

$= \frac{m}{2}(m-1) - m^2 = \frac{m^2}{2} - \frac{m}{2} - m^2$

$= \frac{m}{2}(-m-1) = -\frac{m}{2}(m+1)$