Problem 1 (15 marks).

Suppose that cell phone towers, each of height $h =$ 258 metre, are placed along the Equator of the Earth.

(i) What is the largest distance between two towers measured along the Earth surface such that they can communicate, that is, they are within visual distance from each other?

(ii) What is the minimal number of the towers such that a signal can go from tower to tower around the Earth and come back.

It is assumed that

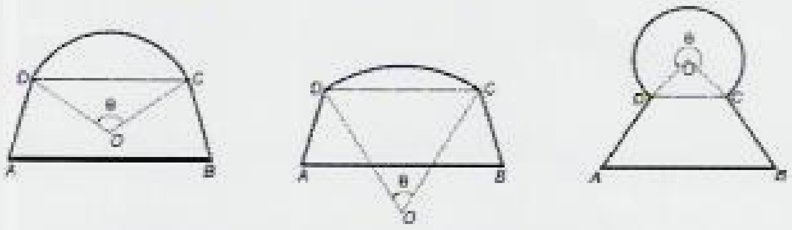
(a) the Earth is a sphere of radius 6378 km;

(b) radio waves propagate along a straight line.

Problem 2 (28 marks).

Window design.

A window is to be made in the shape of a isosceles trapezium topped by an arc of a circle with the length of its respective chord equal to the length of the top base of the trapezium (some possible shapes of windows are presented in Figure below).



The measure of the central angle, θ, that intercepts the arc, is given. In addition, you are given the value of $q = \frac{|AB|}{|CD|}$, the ratio of the lengths of the bottom base of the trapezium to the top one. The following quantity is also known:

$$p = |AB| + |BC| + |AD| - 4s,$$

where s is the length of the arc.

Your values for θ, q and p are found below.

θ (radians) $= \pi/3$ $\qquad q = 4 \qquad$ p (units) $= 1$

(a) Find the dimensions of the window such that the heigh of the trapezium is maximal.

(b) Find the area of the window with these dimensions.