Queen Mary
University of London

Examination by course unit

Date May 2007 Time

DEN108 Dynamics Duration 2 hours 30 minutes

**YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.**

Answer ALL questions from Section A and TWO questions from Section B

**NON-PROGRAMMABLE CALCULATORS ARE PERMITTED IN THIS EXAMINATION. PLEASE STATE ON YOUR ANSWER BOOK THE NAME AND TYPE OF MACHINE USED.**

**COMPLETE ALL ROUGH WORKINGS IN THE ANSWER BOOK AND CROSS THROUGH ANY WORK WHICH IS NOT TO BE ASSESSED.**

**Examiners:**

**C J Lawn**

**A Briggs**

## SECTION A

### Question 1

A tennis player serves the ball at an angle of 5° below the horizontal from a height of 2.50 m, while standing 12 m away from the net. The net is 1.07 m high. Calculate the velocity *v* with which he must serve to clear the net by 5 cm and the distance *s* from the net to the point where the ball hits the ground. (See Figure Q1.) Neglect air resistance and the effect of ball spin.

**[10 marks]**

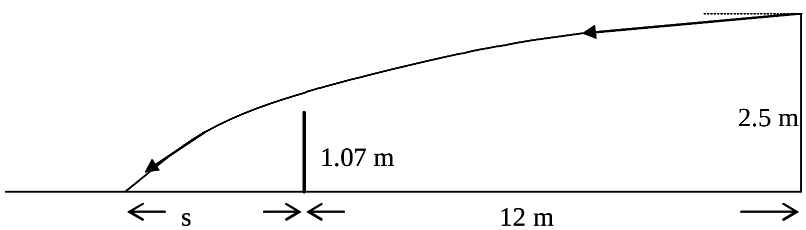


**Figure Q1**

### Question 2

A horizontal pendulum consists of a weightless rod pivoted at one end and attached to a mass *m* at the other. The pendulum is supported by a spring with spring constant *k*. The mass is initially displaced upwards from its horizontal equilibrium position by a small displacement *x*. (See Figure Q2)

a) Obtain an expression for the potential energy stored in the spring.

b) Obtain an expression for the gravitational potential energy of the mass.

c) If the mass is now released, obtain an expression for the kinetic energy of the mass at any instant of time.

d) Hence determine the natural frequency of small oscillations about the equilibrium position if $L$ = 1 m, $a$ = 0.3 m, $m$ = 0.5 kg and $k$ = 500 N/m.

**[10 marks]**

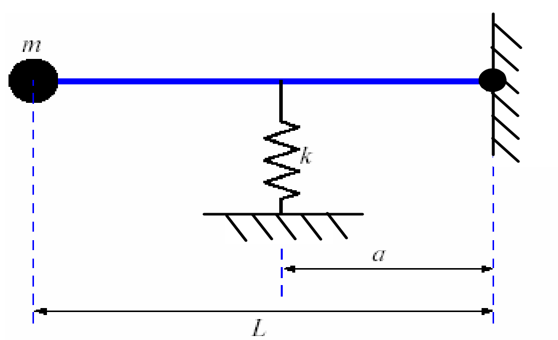


**Figure Q2**

**Question 3**

Derive an expression for the moment of inertia of the thin, right-angle triangular sheet shown in Figure Q3 about the axis Y-Y in terms of its mass, *M* and the dimension *b*.

**[10 marks]**

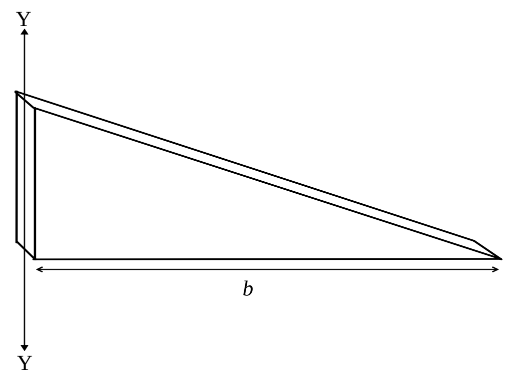


**Figure Q3**

**Question 4**

In Figure Q4, pulleys 1 and 2 are constrained to rotate about their centres. A light cord is wrapped around pulley 1 and its free end is looped around pulley 2 and attached to a body as shown. Pulley 1 has moment of inertia about its centre 5 kg $m^2$ and radius 0.2 m and Pulley 2 has moment of inertia about its centre 10 kg $m^2$ and radius 0.3 m. The suspended body has mass 20 kg.

Use energy methods to calculate the velocity of the suspended body when it has moved a vertical distance 1 m after being released from rest.

**[10 marks]**

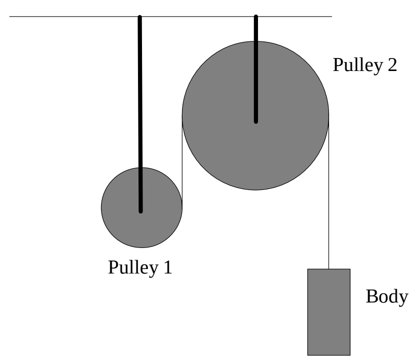


**Figure Q4**

## SECTION B

### Question 5

In the situation illustrated in Figure Q5, the mass $m_1$ is initially at rest. The static friction coefficient between the mass $m_1$ and the slope is given by $\mu_s$, while the kinetic friction coefficient is given by $\mu_k$. It can be assumed that both pulleys are mass-less and that friction in the pulleys can be neglected.

a) Draw a Free Body Diagram of the system.

**[5 marks]**

b) From the Free Body Diagram or otherwise, show that the mass $m_1$ will not move if

$$2m_1(\sin\theta - \mu_s\cos\theta) < m_2 < 2m_1(\sin\theta + \mu_s\cos\theta)$$

**[12 marks]**

c) For a particular case of $\theta$ = 25°, $m_1$ = 30 kg, $m_2$ = 90 kg, $\mu_s$ = 0.30, $\mu_k$ = 0.20, using the Free Body Diagram or otherwise, calculate the acceleration of the mass $m_1$ parallel to the surface on which it rests. Also state whether the mass moves upwards or downwards.

**[13 marks]**

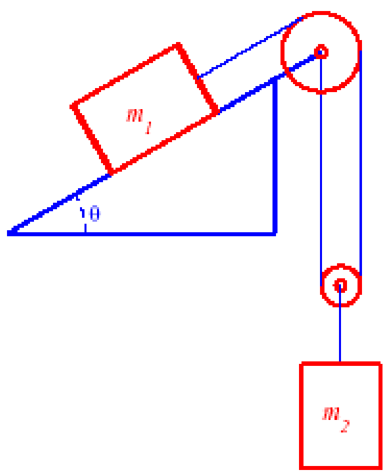


**Figure Q5**

**Question 6**

A rocket R is fired vertically and after a height of 100m has a vertical acceleration given by $a_y = -g - kv^2$, where the last term is due to aerodynamic drag. The drag parameter has a constant value of $k$ = 0.008 $m^{-1}$. The speed of the rocket is $v$ = 20 m/s at the instant when it is at a height of 150 m.

a) Determine the values of $r, \dot{r}, \ddot{r}, \theta, \dot{\theta}, \ddot{\theta}$ from an origin O (the point of observation) 250 m away from the launch point. (See Figure Q6.)

**[20 marks]**

b) Determine the maximum height attained by the rocket.

**[10 marks]**

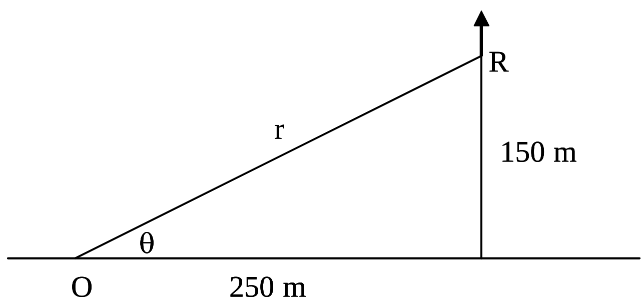


**Figure Q6**

**Question 7**

Part of a winding mechanism is shown in Figure Q7. The combined gear and pulley, B, consists of a pulley of radius $r_1$ mounted rigidly on a gear of radius $r_2$, which in turn is meshed with a second gear, C, of radius $r_3$. A constant clockwise moment $M$ is applied to gear C about its horizontal axis, Q. The moment of inertia of the gear and pulley B about the horizontal axis O is $I$ while the moment of inertia of gear C about Q can be neglected. A rope is wrapped around the pulley and a body, A, of mass $m$ is attached to the free end.

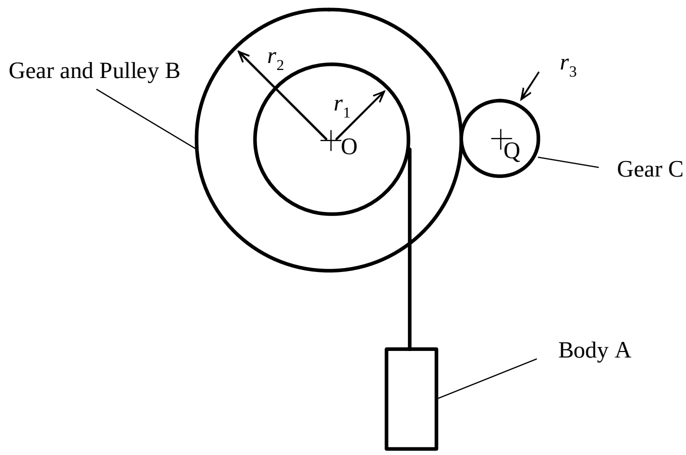


**Figure Q7**

a) Draw a free body diagram of the system

**[6 marks]**

b) Write down the equations of motion for the three parts, A, B and C

**[9 marks]**

c) Hence show that the acceleration of the body A is given by

$$a = \frac{\left( M \dfrac{r_1 r_2}{r_3} - m r_1^2 g \right)}{\left( I + m r_1^2 \right)}$$

where $g$ is the specific force of gravity.

**[15 marks]**

**Question 8**

A circular disc of radius 450 mm has a mass of 300 kg and a radius of gyration about its centre of 300 mm and has a concentric groove of depth 300 mm cut into it as shown. A steady horizontal force *F* of 750 N is applied to a cord wrapped around the groove.

a) Calculate the moment of inertia of the disc about an axis passing through its centre, O and normal to the plane of the figure.

**[3 marks]**

b) Hence calculate the moment of inertia about an axis passing through the point P and normal to the plane of the figure.

**[5 marks]**

c) Assuming no slip between the disc and the surface and hence treating point P as an instantaneous centre of rotation, calculate the acceleration of the centre of mass of the disc as it starts from rest.

(Note: If no slip occurs, the disc will rotate clockwise)

**[11 marks]**

d) Use your results to calculate the minimum value for the coefficient of static friction between disc and surface for which the 'no slip' assumption will be valid.

**[11 marks]**

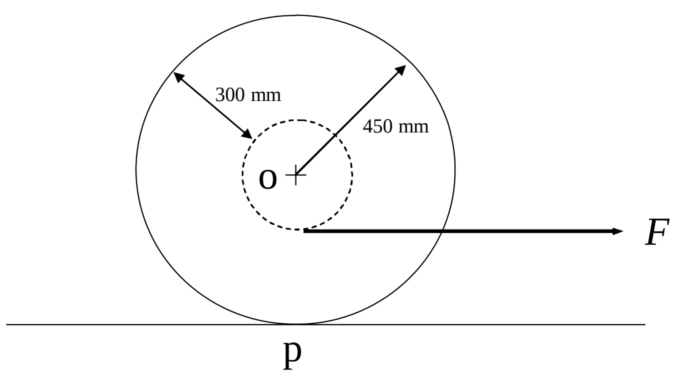


**Figure Q8**

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