2) a) $\forall n, m \quad n > m \Rightarrow a_m > a_n$.

$\exists n, m \quad n > m \Rightarrow a_n > a_m$

b) a) sup 2 max doos not exist.

inf = min = -1

b) ~~min, max do not exist~~

inf = 1, sup = 4

But $\cap \left(\frac{1}{n}, 4 + \frac{1}{n}\right) = (1, 5) \cap \left(\frac{1}{2}, 4\frac{1}{2}\right)$

$\cap \text{---} \left(\frac{1}{n}, 4 + \frac{1}{n}\right) \cap \text{---}$

$= (1, 4]$ $\therefore$ min doos not exist

but max = 4

c) max doos not exist
min doos not exist
inf = 0, sup = $\sqrt[3]{\pi}$

d) sequence $1 - 2\sqrt{1} = -1$
$2 - 2\sqrt{2} = -0.8$
$3 - 2\sqrt{3} = -0.5$

$4 - \cancel{2}\sqrt{4} = 0$

$5 -$

⋮

$100 - 2\sqrt{100} = 80$

Now becomes increasing

No max, sup,

min = inf = -1.

c) Suppose a set S has t[cut off]

max m_1, m_2 and that $m_1 \neq m_2$

and that $m_1, m_2 \in S$.

Suppose $m_1 > m_2$

$s < m_1$ for all $s \in S$

$s < m_2$ for all $s \in S$

but $m_1 > m_2$ $\therefore$ either m_2 no[cut off]

max or $m_1 \notin S$

Contradiction $\therefore$ Only 1 ma[cut off]

3) a) $a_1 = 0$ $a_2 = \frac{1}{3-0} = \frac{1}{3}$

$a_3 = \frac{1}{3-\frac{1}{3}} = \frac{1}{8/3} = 3/8$

$a_3 = \frac{1}{3-3/8} = \frac{1}{21/8} = \frac{8}{21}$

b) $a_1 > a_0$

Suppose $a_n > a_{n-1}$

Prove $a_{n+1} > a_n$

$a_{n+1} = \frac{1}{3-a_n}$, $a_n = \frac{1}{3-a_{n-1}}$

$a_{n+1} - a_n = \frac{1}{3-a_n} - a_n = \frac{1}{3-a_n} - \frac{1}{3-a_{n-1}}$

$= \frac{3-a_{n-1} - 3 + a_n}{(3-a_n)(3-a_{n-1})} = \frac{a_n - a_{n-1}}{(3-a_n)(3-a_{n-1})}$

> 0 since $a_n - a_{n-1} > 0$

Proved.

4) a) yes since $\frac{1}{\sqrt{n}} \to 0$

b) yes since $\frac{1}{n} \to 0$

c) No since $\frac{n+2}{2n} \not\to 0$

d) $\frac{2}{(\ln \frac{1}{n})^n} = \frac{2}{(-\ln n)^n} = \frac{2(-1)^n}{(\ln n)^n}$

$\frac{2}{(\ln n)^n} \to 0$ So Yes.

b) a) $\times$ b) $\checkmark$ if $b=0$

c) $\checkmark$ true automatically

d) $\times$

c) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots$

$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$

Put $x=1$

$\ln(2) = 1 - \frac{1}{2} + \frac{1}{3}$

$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

$\therefore$ ANS $= -\ln 2 = -\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (d)

d) $\dfrac{n! + e^n}{n^n + n^5} = \dfrac{\frac{n!}{n^n} + \left(\frac{e}{n}\right)^n}{1 + \frac{1}{n^{n-5}}}$

as $n \to \infty$ $\quad \dfrac{n!}{n^n} \to 0$

$\dfrac{1}{n^{n-5}} \to 0$

$\left(\dfrac{e}{n}\right)^n \to 0$.

$\therefore \dfrac{n! + e^n}{n^n + n^5} \to \dfrac{0 + 0}{1 + 0} = 0.$

e) a) No eg $a_{n+1} = \dfrac{1}{3 - a_n}$

increasing but bounded by $1/2$.

b) No the limit need not be a member of the sequance

c) No eg $a_n = n$ then limit is infinity.

d) Yes

5) a) $\sum \frac{2^{n+1}}{7^n} + \left(\frac{-3}{7}\right)^n$

$= 2\sum\left(\frac{2}{7}\right)^n + \sum\left(\frac{-3}{7}\right)^n$

$= 2\left(\frac{2/7}{1-2/7}\right) + \frac{(-3/7)}{1-(-3/7)}$

$= \frac{4/7}{5/7} - \frac{3/7}{10/7} = \frac{4}{5} - \frac{3}{10}$

$= \frac{1}{2}$

b) $\sum \frac{3n+1}{n^2+n^3} = \sum \frac{3n+1}{n^2(n+1)}$

$= \sum \frac{2}{n} + \frac{1}{n^2} - \frac{2}{n+1}$

$= \sum \frac{2}{n} - \frac{2}{n+1} + \sum \frac{1}{n^2}$

$= \left(2 - \frac{2}{2}\right) + \left(\frac{2}{2} - \frac{2}{3}\right) + \cdots$

$+ \left(\frac{2}{n} - \frac{2}{n+1}\right) + \sum \frac{1}{n^2}$

$= 2 - \frac{2}{n+1}\Big|_{n=\infty} + \sum \frac{1}{n^2}$

$$\frac{3n+1}{n^2(n+1)} = \frac{a}{n} + \frac{b}{n^2} + \frac{c}{n+1}$$

$$3n+1 = an(n+1) + b(n+1) + cn^2$$

$$1 = b$$

$$-2 = c$$

$$4 = 2a + 2b + c$$

$$= 2a + 2 - c$$

$$a = 2$$

$$\sum \frac{2}{n} - \frac{2}{n+1} + \sum \frac{1}{n^2}$$

$$\underbrace{\sum \frac{2}{n} - \frac{2}{n+1}}_{\text{Telescoping series}} + \sum \frac{1}{n^2}$$

$$= 2 - \frac{2}{n+1}\Big|_{n\to\infty} + \frac{\pi^2}{6} = 2 + \frac{\pi^2}{6}$$

5) a) $\sum \frac{5^n}{n} x^{n+1}$

$a_n = \frac{5^n}{n} x^{n+1}$

$$\frac{a_{n+1}}{a_n} = \frac{\left(5^{n+1} x^{n+2}\right)/n+1}{5^n \; x^{n+1} / n} = \frac{5xn}{n+1}$$

$$-1 < \frac{5xn}{n+1} < 1$$

$$\frac{-(n+1)}{5n} < x < \frac{n+1}{5n}$$

$n \to \infty$

$$-\frac{1}{5} < x < \frac{1}{5}$$

b) $a_n = \frac{1}{n^3} \left(\frac{x}{x+2}\right)^n$

$$\frac{a_{n+1}}{a_n} = \frac{1/(n+1)^3 \left(\frac{x}{x+2}\right)^{n+1}}{1/n^3 \left(\frac{x}{x+2}\right)^n}$$

$$= \left(\frac{n}{n+1}\right)^3 \frac{x}{x+2}$$

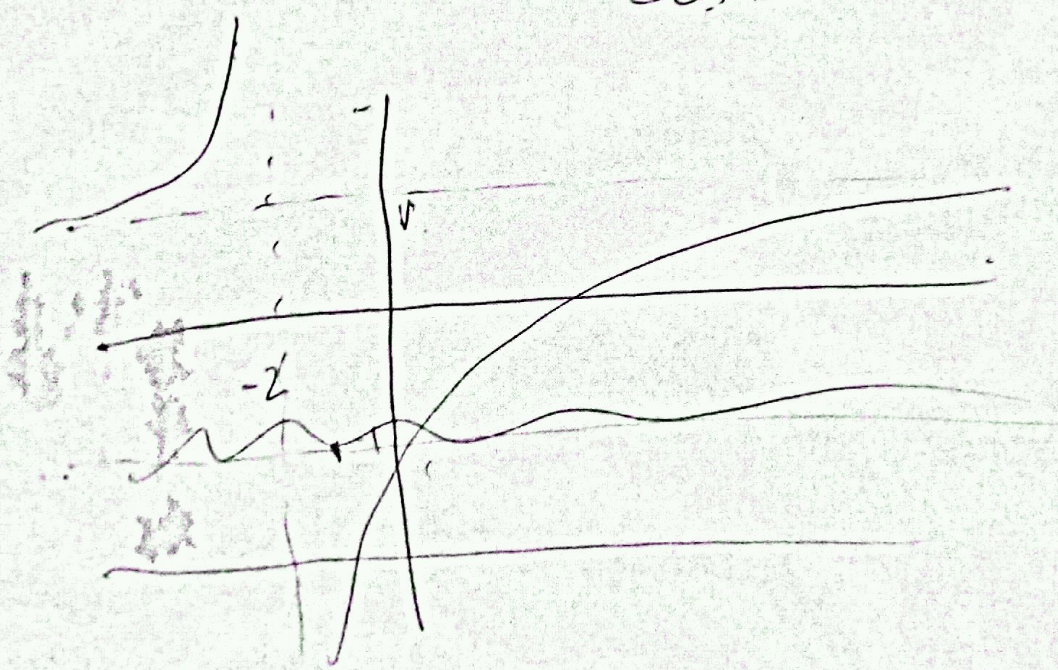
c $-1 < \left(\frac{n}{n+1}\right)^3 \frac{x}{x+2} < 1$

$-\left(\frac{n+1}{n}\right)^3 < \frac{x}{x+2} < \left(\frac{n+1}{n}\right)^3$

$n \to \infty$

$-1 < \frac{x}{x+2} < 1$

$-1 < 1 - \frac{2}{x+2} < 1$



$-1 = 1 - \frac{2}{x+2}$

$+2 = \frac{2}{x+2} \Rightarrow x = -1$

$\underline{\underline{x > -1}}$

c) $\ln(7/6)$

use $\ln(1+x) = \sum \frac{(-1)^{n+1} x^n}{n}$

~~with~~ with $x = 1/6$

For 3 dp need $|a_n| < 0.0005$

$|a_3| = \frac{1/6^3}{3} = \frac{1}{648} > 0.0005$

$|a_4| = \frac{1/6^4}{4} = \frac{1}{5184} < 0.0005$

<u>4 terms</u>