$\frac{a}{b} = \frac{\sqrt{5}}{2} \Rightarrow \frac{a^2}{b^2} = \frac{5}{4} \Rightarrow 4a^2 = 5b^2$

$5 \mid 4a^2 \Rightarrow 5 \mid a^2 \therefore 5 \mid a$ or $a$

$\therefore 5 \mid a \quad \therefore 25 \mid a^2$

Put $a^2 = 25p^2$

Similarly $b^2 = 4q^2$

$\frac{4a^2}{b^2} = 5 \Rightarrow \frac{4 \times 25p^2}{4q^2} = \frac{25p^2}{q^2} = 5$

Contradiction since assumed $\frac{4a^2}{b^2}$ could not cancel.