

①

1) Express z in form $Ae^{i\alpha}$

$$A = |z| = \sqrt{(4\sqrt{3})^2 + 4^2} = 8$$

$$\alpha = \tan^{-1} \left(\frac{4\sqrt{3}}{4} \right) = \frac{\pi}{6}$$

$$z^3 = 8e^{5\pi i/6}$$

$$z_1 = \sqrt[3]{8} e^{(5\pi i/6)/3} = 2e^{5\pi i/18}$$

$$z_2 = \sqrt[3]{8} e^{(5\pi i/6 + 2\pi)/3} = 2e^{17\pi i/18}$$

$$z_3 = \sqrt[3]{8} e^{(5\pi i/6 + 4\pi)/3} = 2e^{29\pi i/18}$$

$$z_1^3 = z_2^3 = z_3^3 = 8e^{5\pi i/6}$$

$$z_1^3 + z_2^3 + z_3^3 = 3(2e^{5\pi i/6})$$

$$z \left(\sum_{n=1}^{\infty} z^n z^{-n} \right) = \frac{z}{1 - z/2}$$

$$\text{Put } z = e^{i\pi/10}$$

$$\text{Then } \lim \left(\sum_{n=1}^{\infty} z^n z^{-n} \right) = \sum_{n=1}^{\infty} 2^{-n} \sin \frac{n\pi}{10}$$

$$= \lim \left(\frac{2}{e^{i\pi/10}} \left(1 - \frac{2}{e^{i\pi/10}} \right) \right) = \frac{2}{1 - e^{i\pi/10}} - 2$$