

$$\lim_{x \rightarrow 0} \left(\frac{1}{\sin^2 x} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0} \left(\frac{x^2 - \sin^2 x}{x^2 \sin^2 x} \right) \quad (7)$$

$$= \lim_{x \rightarrow 0} \left(\frac{2x - 2 \sin x \cos x}{2x \sin^2 x + 2x^2 \sin x \cos x} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{x - \frac{1}{2} \sin 2x}{x \sin^2 x + \frac{1}{2} x^2 \sin 2x} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{\sin^2 x + 2x \sin x \cos x + x \sin 2x + x^2 \cos 2x} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{2 \sin^2 x}{\sin^2 x + 2x \sin 2x + x^2 \cos 2x} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{4 \sin x \cos x}{2 \sin x \cos x + 2 \sin 2x + 4x \cos 2x + 2x \cos 2x - 2x^2 \sin 2x} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{2 \sin 2x}{3 \sin 2x + 6x \cos 2x - 2x^2 \sin 2x} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{4 \cos 2x}{6 \cos 2x + 6 \cos 2x - 4x \sin 2x - 4x \sin 2x - 4x^2 \cos 2x} \right)$$

$$= \frac{4 \times 1}{6 \times 1 + 6 \times 1} = \frac{1}{3}$$

(b) $(1 + 1/x)^x$ take logs

$$\ln (1 + 1/x)^x = x \ln (1 + 1/x)$$

$$= \frac{\ln (1 + 1/x)}{1/x}$$

P.T.O