Section A

1. a) Show that if a sequence of positive real numbers are in a geometric progression then their logarithms are in an arithmetic progression.

(5 marks)

b) The first and last of 8 numbers in a geometric progression are 15 and 84 respectively. Calculate all eight numbers.

(7 marks)

2. Write down the solution to the quadratic

$$x^2 + nx + 1 = 0.$$

x^2

By use of the Binomial Theorem, show that if n is so large that $\frac{1}{n^3}$ can be ignored, then the solutions to the quadratic are

$$\frac{-1}{n} \text{ and } -n + \frac{1}{n}.$$

(12 marks)

3. a) Find the first two terms of the Maclaurin series for $f(x) = e^{\frac{-1}{x^2}}$.

(4 marks)

b) Plot $f(x)$ in a small, open interval (of your choice) around $x = 0$.

(4 marks)

c) Does the answer to part a) of this question represent the function f in any sensible way (very briefly explain your answer)?

(4 Marks)

4. a) (In this question i is the Complex square root of -1). Let $z_1 = 1 - 5i$ and $z_2 = 1 + i$. Find $z_1 - z_2$ and $\frac{z_1 - z_2}{z_2}$.

(5 marks)

b) Use de Moivre's Theorem to find the cube roots of $z_2 = 1 + i$ and illustrate with an Argand diagram.

(7 Marks)

$$-n\left(\frac{1 \pm \sqrt{1 - 4/n^2}}{2}\right)$$

$$= -n\left(\frac{1 \pm (1 - 2/n^2)}{2}\right)$$