7) a) $\int \frac{x}{2x+1}dx = \int \frac{1}{2} - \frac{\frac{1}{2}}{2x+1}dx$

$= \frac{1}{2}x - \frac{1}{4}\ln(2x+1) + C$

b) $\int \frac{dx}{(1-x)(2-x)} = \int \frac{dx}{(x-1)(x-2)} = \int \frac{-1}{x-1} + \frac{1}{x-2}$

$= -\ln(x-1) + \ln(x-2) + C = \ln\left(\frac{x-2}{x-1}\right) + C$

c) $\int_0^{\frac{\pi}{2}} \sin^n x\,dx$

6) $\int \sin^m x\,dx = \int \sin x \sin^{m-1} x\,dx$

$\sin x$: $dv$; $\sin^{m-1} x$: $u$

| $u$ | $v$ |
|---|---|
| $\sin^{m-1} x$ | $-\cos x$ |
| $du$ | $dv$ |
| $(m-1)\sin^{m-2} x \cos x$ | $\sin x$ |

$\int_0^{\frac{\pi}{2}} \sin^m x\,dx = \left[-\sin^{m-1}\cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} (m-1)\sin^{m-2} x \cos^2 x\,dx$

$= 0 + (m-1)\int_0^{\frac{\pi}{2}} \sin^{m-2} x\,dx - (m-1)\int_0^{\frac{\pi}{2}} \sin^m x\,dx$

$\int_0^{\frac{\pi}{2}} \sin^m x\,dx = (m-1)\int_0^{\frac{\pi}{2}} \sin^{m-2} x\,dx - (m-1)\int_0^{\frac{\pi}{2}} \sin^m x\,dx$

$(m-1+1)\int_0^{\frac{\pi}{2}} \sin^m x = (m-1)\int_0^{\frac{\pi}{2}} \sin^{m-2} x\,dx$

$\int_0^{\frac{\pi}{2}} \sin^m x\,dx = \left(\frac{m-1}{m}\right)\int_0^{\frac{\pi}{2}} \sin^{m-2} x\,dx$

8) a) $\int \sin^2 x \cos x\,dx = \int \frac{1}{3}\frac{d}{dx}(\sin^3 x)\,dx$

$= \frac{1}{3}\sin^3 x + C$

b) $\int x \sin x\,dx$ ($x$: $u$; $\sin x$: $dv$)

| $u$ | $v$ |
|---|---|
| $x$ | $-\cos x$ |
| $du$ | $dv$ |
| $1$ | $\sin x$ |

$\int x \sin x\,dx = -x\cos x + \int \cos x\,dx$