Calculus III, 2011: Coursework 1
Will Sutherland.

The deadline is 4PM on Thursday Oct 6th. Hand in the solution for **Question 3 only** *to the green box on the basement floor of the Maths building.*

1. (a) Using the fact that $1/12 = 1/4 - 1/6$, find $\sin \frac{\pi}{12}$,

(b) and use the double angle formulae to find $\cos^2 \frac{\pi}{8}$.

2. Find $\int \sin^4 x \, dx$; hence evaluate $\int_0^{2\pi} \sin^4 x \, dx$.

3. (*) Hand-in:

(a) Prove that $\int_0^{2\pi} \sin mx \cos nx \, dx = 0$ where m, n are positive integers. [5]

(b) Using vectors, calculate the acute angle between any two body diagonals of a cube (note, a body diagonal joins two opposite corners). [5]

4. Describe each of the following surfaces in three dimensional space. [For example, a suitable description of $z = 1$ is 'a plane perpendicular to the z axis' while $x^2 + y^2 = 1$ is 'a circular cylinder of radius 1 whose axis is the z axis'.]

(a) $x^2 + 4y^2 + 9z^2 - 36 = 0$

(b) $x^2 + y^2 - 9z^2 + 36 = 0$

(c) $x^2 + y^2 - 9z + 36 = 0$

[You will find many more examples in Thomas Exercises 12.6. Use them for practice: something like this has been in the exams recently, and has caused some problems.]

5. (Exercise 1.2 of notes). Evaluate, with explanation of the limits, the volume integral

$$\int_V e^x \, dx \, dy \, dz \quad ,$$

where the volume V is the tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$, where a is a positive constant.