Systems Modelling and Simulation

Assignment No 2

Predator Prey Model

By

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Submitted in partial fulfilment of the course requirement to

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Predator Prey Model

Goal:

The goal of this experiment is to model the population dynamics of animals both predator and prey when they are present in an environment .

Description:

1. Initially at time t=0, the population of prey is some value s ay x0 and that of predator is y0
2. Prey eats vegetarian food and predator eats prey
3. If there are no predators, the population of preys will grow exponentially over time and will be growing at the rate of the difference of the birth and death rate.
4. If there are predators, the population of prey will decrease to the extent they interact and the predator is successful in hunting the prey (Alpha)
5. The population of the predator is dependent on the growth rate of the predator due to prey availability (Beta) and the death rate of the predator.

Assumptions:

1. The forest is large and the preys are born and die continuously.
2. There is no shortage of food.

At time t, the prey population is x.
After time t+Δt, the prey population is given by : x+ no of prey born in time Δt- no of natural prey deaths in time Δt.
The rate of growth during interval Δt is : $(x +\Delta x-x)/(t+\Delta t-t)$
Taking lim $\quad \Delta x/\Delta t= dx/dt$.
$\quad \Delta t \rightarrow 0$

Thus we are assuming that the number of preys is large enough to support a continuous death and birth rates.
dx/dt = (birth rate- death rate)*x
$\quad dx/dt= \mu x$
$dx/x = \mu dt$
$\ln(x) = \mu t+\ln(c)$
$\ln(x/c) = \mu t$
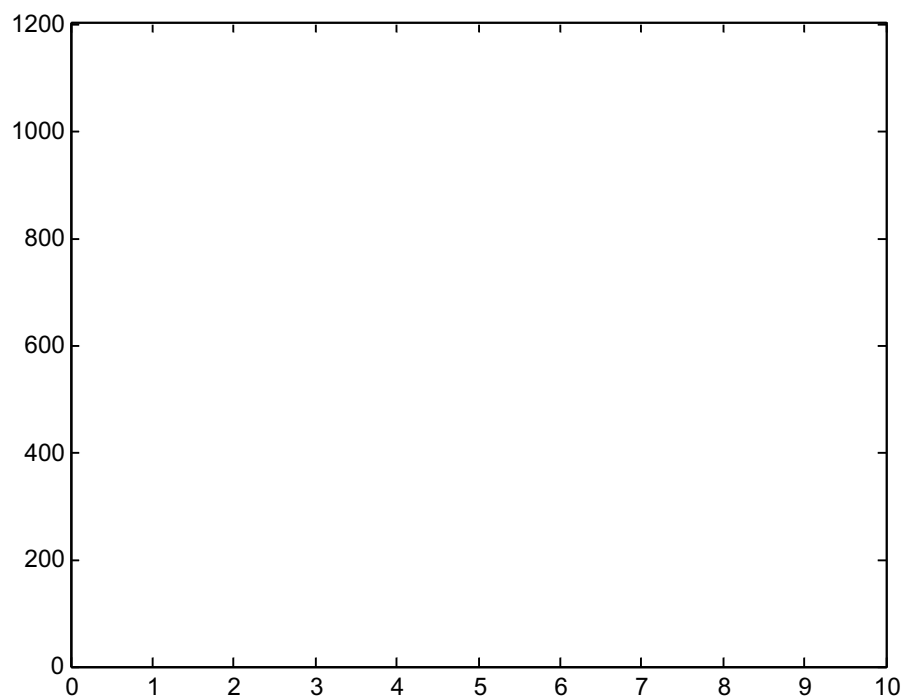$x = x0ce^{\mu t}$ since at t=0, x=x0

Matlab Code:

```
t=0:10;
x0=20;
u= 0.4;
x= x0*exp(u*t)
plot(t,x)
```


However this is not a stable model and we need to cons ider the food shortage due to large prey population. Also we will consider the presence of prey.

$dx/dt = \mu'x - \alpha xy$

$\mu'x$ represents the growth of the deer population

α represents the co efficient of success when predator tries to hunt a prey.

$dy/dt = \beta xy - \lambda y$

β → growth rate of predator due to availability of prey.

λ → starvation due to death due to non -availability of prey

Thus it can be written as

Y= [μ'- αy; -λ+ βx][x;y]'

If μ'=1, λ=1, α=.01, β=.02

Then we can plot the graph for different initial values of prey and predator and find the population model.

Let the initial values of prey and predator be [20 20].

Matlab Code:

Create a file lotka.m with the following code. This will help us use the lotka model with different values of alpha and beta.

MATLAB Code:

```
function yp = lotka(t,y)
%LOTKA   Lotka-Volterra predator-prey model.
global ALPHA BETA
yp = [y(1) - ALPHA*y(1)*y(2); -y(2) + BETA*y(1)*y(2)];
```

type lotka

function yp = lotka(t,y)

%LOTKA   Lotka-Volterra predator-prey model.

global ALPHA BETA

yp = [y(1) - ALPHA*y(1)*y(2); -y(2) + BETA*y(1)*y(2)];

Following code is then used to get the output

t0=0; tfinal =15;

y0 =[20 20]'

global ALPHA BETA

ALPHA =0.01; BETA=0.02;

[t,y] = ode23('lotka',[t0 tfinal],y0);
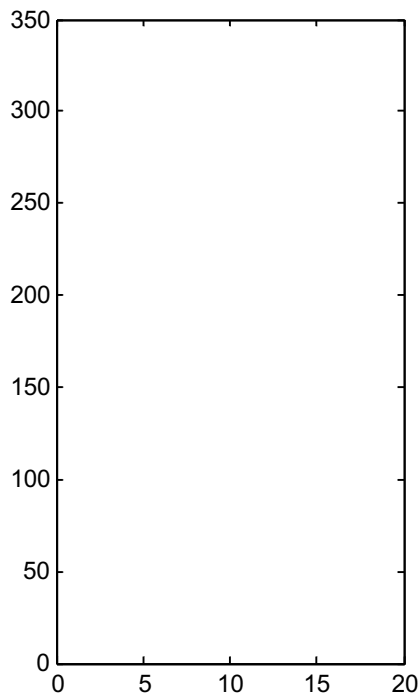
 subplot(1,2,1)

 plot(t,y)

```
title('Time history')

subplot(1,2,2)

plot(y(:,1),y(:,2))

title('Phase plane plot')
```


This shows that population of the predator w ill initially fall and that of prey will fall. However, the population of the predator will then rise owing to availability of prey and the predator population grows before it starts to peak and falter. This is due to lack of availability of prey and the p rey population reaches a low value before it begins to gather pace again. This cycle keeps on repeating indicating that the predator and prey will continue to exist in the system.

If we increase the population of prey to 40, the model shows the following output.

Matlab Code:

```
y0 = [20 40]';

[t,y] = ode23('lotka',[t0 tfinal],y0);

subplot(1,2,1); plot(t,y)
```

title('Time history')

subplot(1,2,2)

plot(y(:,1),y(:,2))

title('Phase plane plot')

The output shows a similar result however the extent to which the population of the predator and prey rises is less in this case. This is because as there are more predator the prey number will not rise to the extent when there are less predators and the predator will find it difficult to gather food.

If again we increase the predator population to 60 the peaks of predator and prey population are lower than previous case and the effect is same as in previous case. The cycle continues to run and the predator and prey continue to exist in the environment.

Matlab Code:

```
y0 = [20 60]';

[t,y] = ode23('lotka',[t0 tfinal],y0);

subplot(1,2,1); plot(t,y)

title('Time history')
```

```
subplot(1,2,2)

plot(y(:,1),y(:,2))

title('Phase plane plot')
```

If we change alpha and beta we can look at how the model changes.

If alpha is made greater than beta then population of prey barely survives. The initial values of prey and predator are restored to 20 and 20.

If α=.05, β=.01,

```
y0 = [20 20]';

 type lotka


function yp = lotka(t,y)

%LOTKA   Lotka-Volterra predator-prey model.

global ALPHA BETA

yp = [y(1) - ALPHA*y(1)*y(2); -y(2) + BETA*y(1)*y(2)];
```
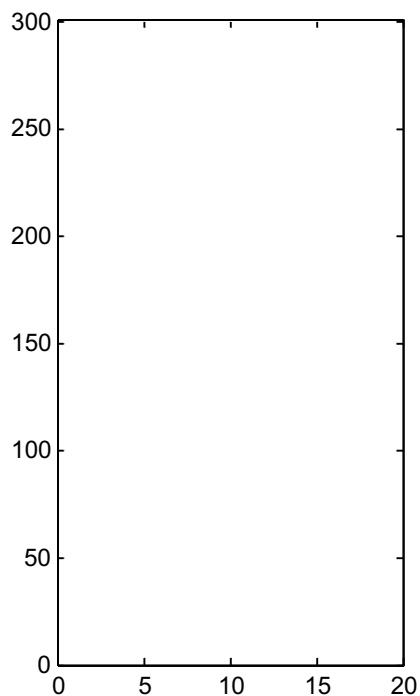
```
ALPHA =0.05;

BETA=0.01;

[t,y] = ode23('lotka',[t0 tfinal],y0);

plot(t,y)

subplot(1,2,1)

 title('Time history')

 subplot(1,2,2)

 plot(y(:,1),y(:,2))

 title('Phase plane plot')
```


From the figure we can see that the prey population reaches a value lower than initial values and the prey population is in single digits. However, the prey still survives and the population once again increases. But the population barely reaches fifty while in the initial case when alpha was less than beta the population is greater than 150. Thus if the interaction between the predator and p rey is high, the prey population will be less.

Now if we make the interaction rate and prey growth rate due to predation equal, the population of both the predator and the prey will reach a high of 400 before decreasing. Thus the interaction rate and preda tion rate decide the behaviour of the system.

ALPHA =0.01;

 BETA=0.01;

 [t,y] = ode23('lotka',[t0 tfinal],y0);

plot(t,y)

subplot(1,2,1)

 title('Time history')
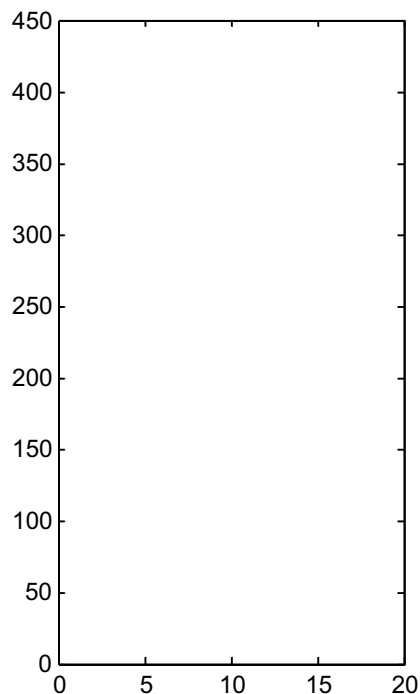
 subplot(1,2,2)

 plot(y(:,1),y(:,2))

 title('Phase plane plot')

Results:

The predator prey model shows that:

1. The values of predator and prey depend on the interaction rate and the growth rate of predator.
2. The model shows that with proper interaction both predator and prey are able to survive. The equilibrium will keep shifting but the process of po pulation increasing and decreasing will keep on repeating. Thus equilibrium in this case will be the repeat of the cycle.