

$$1) a) f(x) = \frac{x^4}{4} - 2x^2$$

$$= \frac{(x^2 - 4)^2}{4} - \frac{(-4)^2}{4} = \frac{(x^2 - 4)^2}{4} - 4$$

$$f(\bar{x}) = -4, f(x) \geq -4 \text{ on } [0, 3]$$

$$f(0) = 0, f(3) = 2.25$$

$$\therefore f([0, 3]) = [-4, 2.25]$$

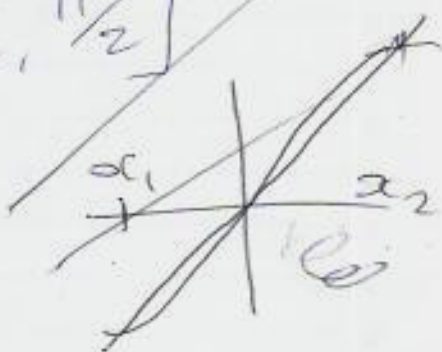
$$b) f([b, c]) \subseteq [b, c]$$

$$\text{Assume } [b, c] \subseteq [-\pi/2, \pi/2]$$

Assume intersect at x_1

$$\therefore \sin ax_1 = x_1$$

$$a \cos ax_1 = 1$$



$$c) |\ln b - \ln a| \leq |b - a|$$

$a \geq 1$ fixed

$$|\ln 1 - \ln a| = |0 - \ln a| \leq |\ln a|$$

$$|\ln a| \leq |a|$$

$$\text{Since } (\ln a)' = \frac{1}{a} \leq 1 \text{ for } a \geq 1$$

ditto for b fixed
 \therefore true for a, b both