

$$3) 2^{(-1)^n n} = 2^n$$

$$n \text{ even } |2^{(-1)^n n}| = |2^n| = 2^n$$

$$\text{as } n \text{ even} \rightarrow \infty, 2^n \rightarrow \infty$$

$$\therefore \sup = \infty \text{ (or no finite sup.)}$$

$$n \text{ odd, } |2^{(-1)^n n}| = |2^{-n}| = \frac{1}{2^n}$$

$$\text{as } n \text{ odd} \rightarrow \infty, \frac{1}{2^n} \rightarrow 0$$

$$\therefore \infimum = 0$$

b) Suppose not, ~~so~~ and suppose $M_2 > M_1$

$$\text{Then } M_2 = M_1 + \epsilon, \epsilon > 0$$

~~M_1 and M_2 are~~

$$M_1 \text{ a supremum} \Rightarrow x \leq M_1 \text{ for all } x$$

$$\text{then } x < M_2 - \epsilon \text{ for all } x$$

$$\text{and } M_2 \text{ not a supremum}$$

$$\text{contradiction} \therefore M_1 = M_2$$

$$c) a_n = 1 - \frac{1}{n}, \sup a_n = 1$$

$$b_n = \frac{1}{n}, \sup b_n = 1$$

$$a_n + b_n = 1 \text{ but } \sup a_n + \sup b_n = 2$$

$$\therefore \sup(a_n + b_n) = 1 \neq \sup a_n + \sup b_n = 2$$