$\sum \left(\frac{k}{k+1}\right)^k$

$\lim_{k\to\infty} \left(\frac{k}{k+1}\right)^k = \lim_{k\to\infty}\left(\frac{1}{1+1/k}\right)^k = \lim_{k\to\infty}\frac{1}{(1+1/k)^k}$

$= \frac{1}{e} > 0$

$\therefore$ terms do not tend to zero

so sum tends to ∞

$\sum\left(\frac{k}{k+1}\right)^{k^2} = \sum\left(\left(\frac{k}{k+1}\right)^k\right)^k$

As before $\left(\frac{k}{k+1}\right)^k \to \frac{1}{e}$

$\therefore$ there exists N such that for $n>N$

$\left(\frac{k}{k+1}\right)^k < \frac{2}{e}$

$\therefore \sum_{k=1}^{\infty}\left(\left(\frac{k}{k+1}\right)^k\right)^k \le \sum_{k=1}^{N}\left(\frac{k}{k+1}\right)^{k^2} + \sum_{k=N+1}^{\infty}\left(\frac{2}{e}\right)^k$

the second series has $\left(\frac{2}{e}\right) < 0.7 < 1$

$\therefore$ convergent and the first has finite number of terms

$\therefore$ Bounded $\therefore \sum\left(\frac{k}{k+1}\right)^{k^2}$ converges