So $a_{n+1} = \dfrac{a_{n-1}}{n(n+1)}$

$a_0 = A$, $A_2 = \dfrac{A_0}{1\times 2} = \dfrac{A_0}{2!}$, $A_4 = \dfrac{A_2}{3\times 4} = \dfrac{A_0}{1\times 2\times 3\times 4} = \dfrac{A_0}{4!}$

in general, n even, $A_n = \dfrac{A_0}{4!}$

n odd $a_{n+1} = \dfrac{a_{n-1}}{n(n+1)}$

$a_1 = B$ $a_3 = \dfrac{B}{2\times 3} = \dfrac{B}{3!}$

$a_5 = \dfrac{B}{4\times 5} = \dfrac{B}{1\times 2\times 3\times 4\times 5} = \dfrac{B}{5!}$

In general $a_n = \dfrac{B}{n!}$ n odd.

$$y = \sum_{n \text{ even}} a_n x^n + \sum_{n \text{ odd}} b_n x^n$$

$$= \sum_{n \text{ even}} \frac{A}{n!} x^n + \sum_{n \text{ odd}} \frac{B}{n!} x^n$$

$$= A \sum_{n \text{ even}} \frac{1}{n!} x^n + B \sum_{n \text{ odd}} \frac{1}{n!} x^n$$

$$= A \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n} + B \sum_{n=0} \frac{1}{(2n+1)!} x^{2n+1}$$