1) $\sum_{n=1}^{\infty} 2^n x^{n+1} = \sum_{n=2}^{\infty} 2^{n-1} x^n$

$\sum_{n=0}^{\infty}(n+1)x^n = (0+1)x^0 + (1+1)x^1 + \sum_{n=2}^{\infty}(n+1)x^n$

$= 1 + 2x + \sum_{n=2}^{\infty}(n+1)x^n$

$\therefore -\sum_{n=1}^{\infty} 2^n x^{n+1} + \sum_{n=0}^{\infty}(n+1)x^n = 1 + 2x + \sum_{n=2}^{\infty}(2^{n-1} + n + 1)x^n$

$f(x) = 1 + 2x$

$g(n) = 2^{n-1} + n + 1$

2) $y = \sum_{n=0}^{\infty} a_n x^n$, $\frac{dy}{dx} = \sum_{n=1}^{\infty} a_n \times n x^{n-1}$

$= \sum_{n=0}^{\infty} a_{n+1} \times (n+1) x^n$

$\therefore \frac{dy}{dx} - y = 0$ becomes

$\cancel{\sum_{n=0}^{\infty} u(n) x^n} - \sum_{n=0}^{\infty} a_{n+1}(n+1)x^n - \sum_{n=0}^{\infty} a_n x^n = 0$

Write as one summation

$\sum_{n=0}^{\infty} (a_{n+1}(n+1)x^n - a_n x^n) = 0$

$\sum_{n=0}^{\infty} (a_{n+1}(n+1) - a_n) x^n = 0$

$u(n) = a_{n+1}(n+1) - a_n$

b) true for all x^n so $u(n) = 0$

for all n $\therefore a_{n+1}(n+1) - a_n = 0$

$\therefore a_{n+1} = \frac{a_n}{n+1}$

c) $y = ce^x$