$$2a_2 + 6a_3x + \sum_{n=2}^{\infty}(n+2)(n+1)a_{n+2}x^n$$

$$-\sum_{n=2}^{\infty} n(n-1)a_n x^n - 2a_1 x + \sum_{n=2}^{\infty} -2na_n x^n$$

$$+\alpha(\alpha+1)a_0 + \alpha(\alpha+1)a_1 x + \sum_{n=2}^{\infty}\alpha(\alpha+1)a_n x^n = 0$$

$$(2a_2 + \alpha(\alpha+1)a_0) + x(6a_3 - 2a_1 + \alpha(\alpha+1)a_1)$$

$$+ \sum_{n=2}^{\infty}((n+2)(n+1)a_{n+2} - 2na_n + \alpha(\alpha+1)a_n)x^n = 0$$

set all coefficients equal zero

$$2a_2 + \alpha(\alpha+1)a_0 = 0 \Rightarrow a_2 = -\frac{\alpha(\alpha+1)}{2}a_0$$

$$6a_3 - 2a_1 + \alpha(\alpha+1)a_1 = 0 \Rightarrow a_3 = \frac{(2 - \alpha(\alpha+1))}{6}a_1$$

$$(n+2)(n+1)a_{n+2} - 2na_n + \alpha(\alpha+1)a_n = 0$$

$$a_{n+2} = \frac{2na_n - \alpha(\alpha+1)a_n}{(n+2)(n+1)}$$

$$= \frac{(2n - \alpha(\alpha+1))}{(n+2)(n+1)}a_n$$