4) $y = \sum_{n=0}^{\infty} a_n x^n \qquad \frac{dy}{dx} = \sum_{n=1}^{\infty} n a_n x^{n-1}$

$$\frac{d^2y}{dx^2} = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$(1-x^2)\frac{d^2y}{dx^2} = (1-x^2)\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$= \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1) a_n x^n = \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n - \sum_{n=2}^{\infty} n(n-1)a_n x^n$$

$$-2x\frac{dy}{dx} = \sum_{n=1}^{\infty} -2n\, a_n x^n = -2a_1 x + \sum_{n=2}^{\infty} -2n a_n x^n$$

Equation becomes

$$\cancel{\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_n x^n + \sum_{n=2}^{\infty} -2n a_n x^n - 2a_1 x + \sum_{n=2}^{\infty} \alpha(\alpha+1)a_n x^n + \alpha(\alpha+1)a_0 + \alpha(\alpha+1)a_1 = 0}$$

~~combine summations as one~~

$$\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n - \sum_{n=2}^{\infty} n(n-1)a_n x^n$$

$$-2a_1 x + \sum_{n=2}^{\infty} -2n a_n x^n + \sum_{n=0}^{\infty}\alpha(\alpha+1)a_n x^n = 0$$

separate terms up to $n=2$ in each summation