

Ordinary Differential Equations - MA2020  
Course Work I (Mark Joy)

Hand-in date: 8th March 2011, by 1p.m. in student office.

Please note that **all** your answers should be **clear, concise and accurate**. Your arguments must be clearly presented and follow logically. More marks will be allotted to concise (and obviously accurate) answers. If I can't read your handwriting or find it impossible to follow your arguments then you risk scoring no marks.

**Q 1** A free-falling sky-diver of mass  $M$  jumps from an aeroplane and before he opens his parachute experiences air resistance which is proportional to the **square** of the magnitude of his velocity.

- a) Show that the equation of motion for the sky-diver can be written as

$$\frac{dv}{dt} = -g + \frac{k}{M}v^2, \quad (1)$$

where  $v$  is the magnitude of the sky-diver's velocity,  $k$  is the proportionality constant for the air resistance and  $g$  is the gravitational constant.

[5 marks]

- b) What initial data concerning,  $v(0)$ , is consistent with the following solution the of differential equation (1) obtained in part (a) of this question,

$$v(t) = \sqrt{\frac{Mg}{k} \frac{(e^{-At} - 1)}{(e^{-At} + 1)}}, \quad (2)$$

where

$$A = 2\sqrt{\frac{gk}{M}}.$$

[15 marks]

- c) Using the expression (2) for the velocity obtained in part (b) of this question, show that there exists a limiting or terminal velocity,  $v_L$ , such that

$$v(t) \rightarrow v_L, t \rightarrow \infty.$$

What is  $v_L$ ?