Ordinary Differential Equations - MA2020B

MaRKpjoy

All assignments must be handed-in to the student office in the Sopwith building by **1.00.p.m. on Monday 9th March 2009**.

> Please note that **all** your answers should be **clear**, **concise** and **accurate** justifications/proofs.

Q 1. A particle of mass M is hurled directly upwards with an initial speed of v_0. If the coefficient of air resistance is $\beta > 0$ show that

a) the object reaches a maximum height of

$$\frac{Mv_0}{\beta} - \frac{M^2 g}{\beta^2} \log(1 + \frac{\beta v_0}{Mg})$$

15 marks

b) the time taken to each this height is

$$\frac{M}{\beta} \log(1 + \frac{\beta v_0}{Mg}).$$

15 marks

Q 2. If you plot the curves given by the equation, $y = x^2 + c$, for differing values of the constant c, you get a *family* of curves in the plane. (Try it in your favourite software package)

If two families of curves cross at right angles everywhere they are called *orthogonal families*. **By deriving an appropriate ordinary differential equation**, find the equation, $x = x(y)$, of the family of curves orthogonal to the family $y = x^2 + c$. (Hint: if

$$\begin{pmatrix} x \\ y(x) \end{pmatrix} \tag{1}$$

is the position vector of a curve given by the equation $y = y(x)$, then

$$\begin{pmatrix} 1 \\ y'(x) \end{pmatrix} \tag{2}$$

is tangent to the curve).

30 marks

Q 3. Consider the ordinary differential equation

$$\frac{dx}{dt} = x^3,$$

by separation of variables find all solutions of this equation satisfying $x(t) \neq 0$, explicitly stating the domain of definition of the solution. What happens if $x(t) = 0$?

15 marks