Question 5 (Revision)

Consider the steady, axially symmetric flow of a viscous fluid (coefficient of viscosity μ, density ρ) along a long, straight pipe of uniform circular cross-section of radius a, that is inclined at a fixed angle α to the horizontal (see Figure 4). The pipe is stationary, and gravity is the only body force acting on the fluid.

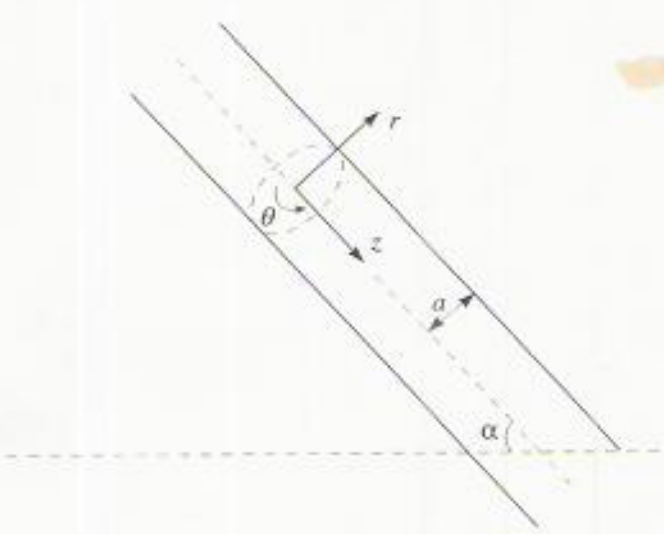

Figure 4

(i) Write down the boundary conditions at the surface of the pipe, $r = a$, when (r, θ, z) are cylindrical polar coordinates as indicated in Figure 4. [$\frac{1}{2}$]

(ii) Starting from equations in cylindrical polar coordinates (with the z-axis as shown in Figure 4) and stating your simplifications carefully, show that the velocity component u_z, away from the ends of the pipe, is given by

$$u_z = \frac{C}{4\mu}\left(a^2 - r^2\right),$$

in which $C = -\frac{d}{dz}(p - \rho\Omega)$ where p is the fluid pressure and Ω is the body force potential. [$13\frac{1}{2}$]

(iii) Hence show that the Hagen–Poiseuille formula for this inclined pipe flow is given by

$$Q = \frac{\pi a^4}{8\mu}\left(-\frac{dp}{dz} + \rho g \sin\alpha\right),$$

where g is the magnitude of the acceleration due to gravity. [2]

(iv) Water flows downward at a rate of $6 \times 10^{-5}\,\mathrm{m^3\,s^{-1}}$ through a vertical pipe with a radius of $0.05\,\mathrm{m}$. What will be the pressure gradient in the flow, and how does it compare with the pressure gradient if the pipe is horizontal? Interpret your results *briefly*.

[Take $g = 9.8\,\mathrm{m\,s^{-2}}$.] [4]