Your attention is drawn to Section 9.6 of the *Student Handbook*, which includes the following phrase: '... no extension to the cut-off date will normally be allowed for a course's last assignment'.

This assignment covers *Units 13* and *14*, as well as revision of the whole course. Questions 1 and 2 are allotted 25 marks each, Questions 3 and 5 are allotted 20 marks each, and Question 4 is allotted 10 marks.

Question 1 (Unit 13)

(i) Decide which of the following functions represent possible solutions of the one-dimensional wave equation, stating the appropriate value of the wave speed, c, in each case. For those functions which are solutions, state whether they represent right-propagating waves, left-propagating waves, a linear combination of right- and left-propagating waves, or standing waves.

(a) $u(x,t) = \exp(\cos(2x - t))$

(b) $u(x,t) = (x^2 + t^2)^{1/2}$

(c) $u(x,t) = (x - t)^{3/2}$

(d) $u(x,t) = 5\cos(7x)\sin(7t)$ [5]

(ii) A uniform infinite elastic string, on which waves can propagate with speed $c = 1\,\mathrm{m\,s^{-1}}$, occupies the x-axis when in its undisturbed state. The displacement of the string in the subsequent motion is described by the function $u(x,t)$ which satisfies the one-dimensional wave equation. Initially,

$$u(x,0) = \begin{cases} 1, & x \geq 1, \\ 0, & -1 \leq x < 1, \\ -1, & x < -1, \end{cases}$$

and the string is released from rest at $t = 0$.

Draw a diagram like Figure 5 on page 26 of *Unit 13* for this situation, and use d'Alembert's solution to find the displacement of the string as a function of x and t in each of the six regions bounded by characteristics and the x-axis, and on the bounding characteristics themselves.

Draw the (x,t)-diagram for the subsequent motion of the string, showing appropriate characteristics and the values of the displacement in each of the six regions and on the characteristics.

Sketch the string profiles for $t = 0$, $t = 1$ and $t = 2$. What is the nature of the initial discontinuities in this problem, and what happens to them as time progresses? [14]

(iii) A uniform semi-infinite elastic string occupies the positive x-axis when in equilibrium. Let the displacement of the string in the subsequent motion be given by $u(x,t)$, which satisfies the one-dimensional wave equation with wave speed c. Initially, the displacement and the velocity of the string are given by

$$u(x,0) = 1 - \cos x \quad (x > 0) \qquad \text{and} \qquad \frac{\partial u}{\partial t}(x,0) = \sin x \quad (x > 0).$$

Specify a problem involving the motion of an infinite string which would assist you in determining the motion of the semi-infinite string for each of the following cases (but do *not* solve the problems):

(a) the end $x = 0$ is fixed;

(b) the end $x = 0$ is free. [6]