

This assignment covers *Units 9 to 12*. Each question is allotted 25 marks.

**Question 1** (Unit 9)

This question concerns the differential equation

$$K \frac{\partial^2 u}{\partial x^2} + (K+3) \frac{\partial^2 u}{\partial x \partial y} - 3 \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + 3 \frac{\partial u}{\partial y} = 0,$$

in which  $K$  is a real constant.

- (i) For which values of  $K$  is the equation

(a) parabolic, (b) hyperbolic, (c) elliptic? [4]

The remaining parts of this question concern the case when  $K = 2$ . You are now asked to use the method of characteristics to solve the differential equation.

- (ii) Show that

$$\zeta = y - 3x, \quad \phi = y + \frac{1}{2}x$$

are appropriate characteristic coordinates. [3]

- (iii) Use these characteristic coordinates to reduce the differential equation to

$$7 \frac{\partial^2 u}{\partial \zeta \partial \phi} - \frac{\partial u}{\partial \phi} = 0. \quad [7]$$

- (iv) Hence show that the general solution is

$$u(x, y) = g(y - 3x) + e^{(y-3x)/7} h\left(y + \frac{1}{2}x\right),$$

where  $g$  and  $h$  are arbitrary functions. [4]

- (v) Determine the particular solution that satisfies the two conditions

$$u(x, 0) = e^{-3x/7}$$

and

$$\frac{\partial u}{\partial y}(x, 0) = e^{-3x/7}. \quad [7]$$

**Question 2** (Unit 10)

- (i) (a) Show that the function  $g$  defined by

$$g(x) = |e^x - e^{-x}| \quad (-\pi < x < \pi)$$

is an even function. [2]

- (b) Hence, or otherwise, determine the Fourier series on the interval  $-\pi < x < \pi$  for the function  $f$  defined by

$$f(x) = \sin 3x + |e^x - e^{-x}|. \quad [11]$$

- (ii) Determine the first five non-zero terms of the power series solution about  $x = 0$  of the differential equation

$$(4 - x^2) \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + 5y = 0$$

such that

$$y(0) = 8 \quad \text{and} \quad \frac{dy}{dx}(0) = 0.$$

Determine an interval of convergence for this power series solution. [12]