Question 2 (Unit 6)

(a) Crystal Rapids is a section of the Grand Canyon river where the (approximately) 100 m broad river narrows to a 'throat' of 25 m and then returns to its 100 m breadth. It may be modelled by an open flat-bottomed channel of breadth $b(x)$ along which the river, an inviscid, incompressible fluid of depth $h(x)$, flows steadily, as shown in Figure 2. In the figure, the x-axis is in the direction of the flow, which is to the right.

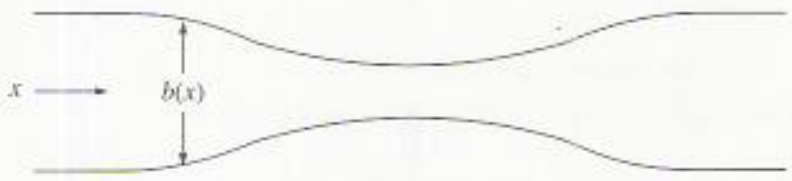

Figure 2

(i) By using Bernoulli's Equation, show that the specific energy function,

$$E = \frac{V^2}{2gh^2b^2} + h,$$

where V is the volume flow rate and g is the magnitude of the acceleration due to gravity, is constant. [3]

(ii) Upstream of the throat, the volume flow rate is measured as $283\,\mathrm{m^3\,s^{-1}}$ ($\simeq 10\,000$ cubic feet per second) and the depth as 4.5 m. Show that the depth of water at the throat, h metres, satisfies

$$h^3 - 4.52h^2 + 6.41 = 0,$$

where the last two coefficients are given to three significant figures. Hence determine the depth and speed of the flow at the throat. Find the Froude numbers in the upstream and throat sections, and hence confirm that for subcritical flow upstream, the depth of flow is reduced at the throat but the speed increases there.

[Take $g = 10\,\mathrm{m\,s^{-2}}$.] [12]

(b) A fluid flows in a horizontal pipe with a contraction in which the cross-sectional area decreases to $\frac{1}{4}$ of its original value. The original speed is u_0 and the original pressure is p_0. By making several simplifying assumptions, show that the speed and pressure in the contraction, u_c and p_c respectively, satisfy

$$u_c = 4u_0$$

and

$$p_c = p_0 - 7.5\rho u_0^2,$$

where ρ is the density of the fluid. List your assumptions. Explain, briefly, why the model cannot be used if $p_0 < 7.5\rho u_0^2$. [5]