for $\omega_1 = \sqrt{\frac{k}{m}}$, substituted into matrix ①.

$$\begin{bmatrix} k^2/m^2 - 5k^2/m^2 & 4k/m \\ 4k/m & k/m - 5k/m \end{bmatrix} = \begin{bmatrix} -4k/m & 4k/m \\ 4k/m & -4k/m \end{bmatrix}$$

The eigenvectors are the solns to $-4x_1 + 4x_2 = 0$ ✓
ie $(1\ 1)^T$ the normal mode displacement ratio is 1. ✓ 3/3

for $\omega_2^2 = 3\sqrt{\frac{k}{m}}$

$$\begin{bmatrix} 9k/m - 5k/m & 4k/m \\ 4k/m & 9k/m - 5k/m \end{bmatrix} = \begin{bmatrix} 4k/m & 4k/m \\ 4k/m & 4k/m \end{bmatrix}$$

The eigenvectors are the solns to $4x_1 + 4x_2 = 0$ ✓
ie $(1\ -1)^T$, the normal mode displacement ratio is -1. ✓

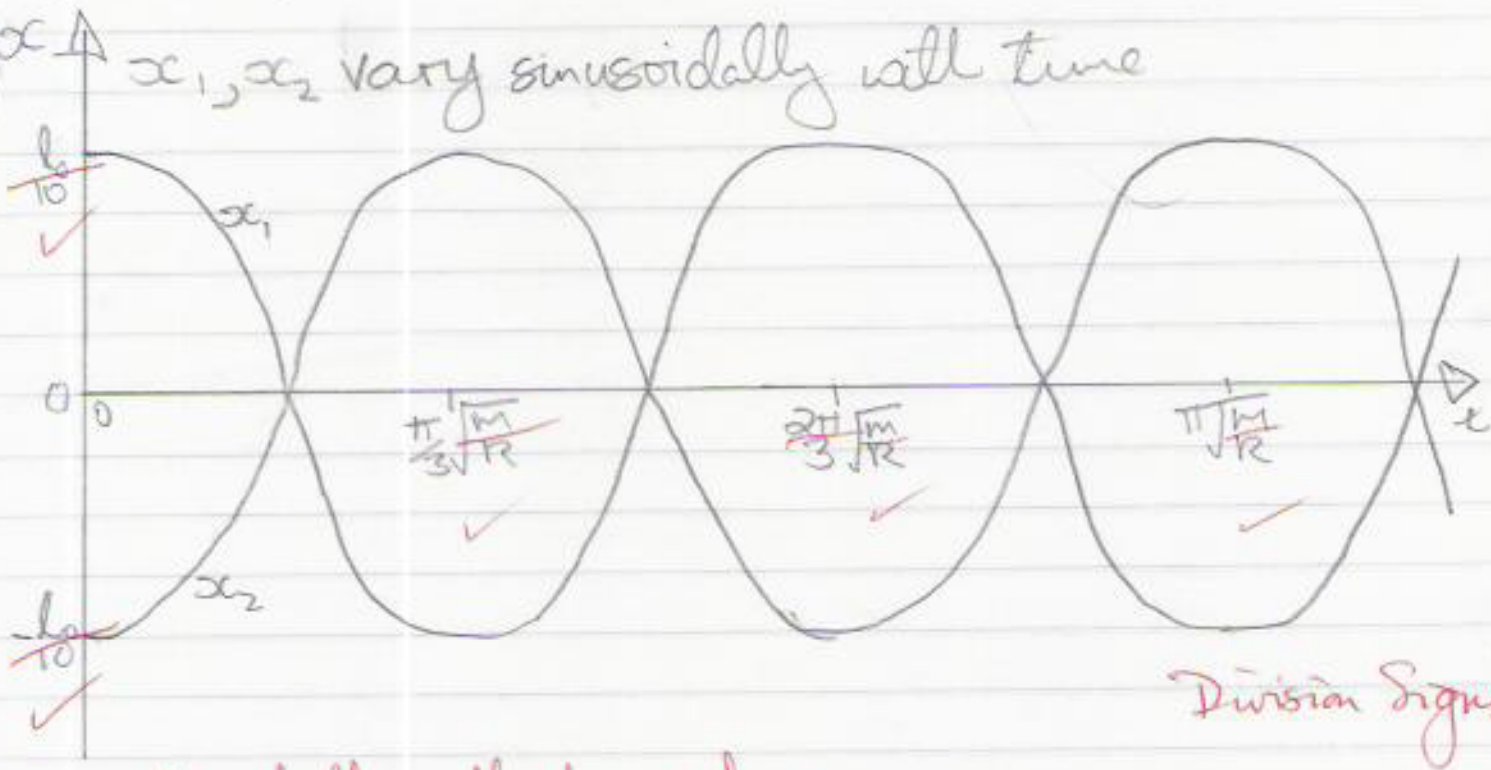
iv) $x_1 = A\cos t\sqrt{\frac{k}{m}} + B\sin t\sqrt{\frac{k}{m}} + C\cos 3t\sqrt{\frac{k}{m}} + D\sin 3t\sqrt{\frac{k}{m}}$ ✓ 2/2

$x_2 = A\cos t\sqrt{\frac{k}{m}} + B\sin t\sqrt{\frac{k}{m}} - C\cos 3t\sqrt{\frac{k}{m}} - D\sin 3t\sqrt{\frac{k}{m}}$ ✓

or equivalently 3/3

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\left(A\cos t\sqrt{k/m} + B\sin t\sqrt{k/m}\right) + \begin{pmatrix} 1 \\ -1 \end{pmatrix}\left(C\cos 3t\sqrt{k/m} + D\sin 3t\sqrt{k/m}\right)$$ ✓

v)



x_1, x_2 vary sinusoidally with time ✓

Division Signs!

You should really have shown me the working out to get this — however B.O.D. 1/1 + 2/2 for diagram.