Your notation has been quite good up to this point!

6) $\underline{F}_1 = x^3\underline{i} + x^2y^2\underline{j} + xyz\underline{k}$

$\underline{F}_2 = y(2x+z)\underline{i} + x(x+z)\underline{j} + xy\underline{k}$

For a conservative field $\underline{F}$, curl $\underline{F} = \underline{0}$ ✓ 1/1

Curl $\underline{F}_1 = \left(\frac{\partial F'_y}{\partial z} - \frac{\partial F'_z}{\partial y}\right)\underline{i} + \left(\frac{\partial F'_z}{\partial x} - \frac{\partial F'_x}{\partial z}\right)\underline{j} + \left(\frac{\partial F'_x}{\partial y} - \frac{\partial F'_y}{\partial x}\right)\underline{k}$ Incorrect formula. This is − curl $\underline{F}$. Refer to the Handbook P. 4 2

Where $\underline{F}_1 = (F'_x, F'_y, F'_z) = F'_x\underline{i} + F'_y\underline{j} + F'_z\underline{k}$

Curl $\underline{F}_1 = \left(\frac{\partial(x^2y^2)}{\partial z} - \frac{\partial(xyz)}{\partial y}\right)\underline{i} + \left(\frac{\partial(xyz)}{\partial x} - \frac{\partial(x^3)}{\partial z}\right)\underline{j} + \left(\frac{\partial(x^3)}{\partial y} - \frac{\partial(x(x+z))}{\partial x}\right)\underline{k}$ incorrect value of F_y

$= (0 - xz)\underline{i} + (yz - 0)\underline{j} + (0 - 2x + z)\underline{k}$ 1/3

$= -xz\underline{i} + yz\underline{j} - (2x+z)\underline{k} \neq 0$!

Hence $\underline{F}_1$ is not conservative

Curl $\underline{F}_2 = \left(\frac{\partial F^2_y}{\partial z} - \frac{\partial F^2_z}{\partial y}\right)\underline{i} + \left(\frac{\partial F^2_z}{\partial x} - \frac{\partial F^2_x}{\partial z}\right)\underline{j} + \left(\frac{\partial F^2_x}{\partial y} - \frac{\partial F^1_y}{\partial x}\right)\underline{k}$ ditto

$= \left(\frac{\partial(x^2+xz)}{\partial z} - \frac{\partial(xy)}{\partial y}\right)\underline{i} + \left(\frac{\partial(xy)}{\partial x} - \frac{\partial(y(2x+z))}{\partial z}\right)\underline{j} + \left(\frac{\partial(y(2x+z))}{\partial y} - \frac{\partial(x(x+z))}{\partial x}\right)\underline{k}$

$= (x - x)\underline{i} + (y - y)\underline{j} + (2x+z-2x-z)\underline{k}$

$= \underline{0}$ vector zero

Hence $\underline{F}_2$ is conservative ✓ 2/3

ii) $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$

$= t^2\underline{i} + t\underline{j} + t^3\underline{k}$

so $\frac{d\underline{r}}{dt} = 2t\underline{i} + \underline{j} + 3t^2\underline{k}$ ✓ 3/3

and $\underline{F}_1 = x^3\underline{i} + x^2y^2\underline{j} + xyz\underline{k}$

$= (t^2)^3\underline{i} + (t^2)^2(t)^2\underline{j} + (t^2)(t)(t^3)\underline{k}$

$= t^6\underline{i} + t^6\underline{j} + t^6\underline{k}$ ✓

$\int_C \underline{F}\cdot d\underline{r} = \int_0^1 (t^6\underline{i} + t^6\underline{j} + t^6\underline{k})\cdot\frac{d\underline{r}}{dt}\,dt$ vector notation

$= \int_0^1 (t^6\underline{i} + t^6\underline{j} + t^6\underline{k})\cdot(2t\underline{i} + \underline{j} + 3t^2\underline{k})\,dt$

$= \int_0^1 (2t^7 + t^6 + 3t^8)\,dt$ ✓

$= \left[\frac{t^8}{4} + \frac{t^7}{7} + \frac{t^9}{3}\right]_0^1$ ✓