$\phi(x,y,z) = x^2y + xyz + \phi(x) + \phi(xz) + \phi(z) + K$ ②

Why should the 3 integrations all use "K"?

$\frac{\partial\phi}{\partial z} = xy$

$\int \partial\phi = \int xy\,\partial z$

$\phi(x,y,z) = xyz + \phi(x) + \phi(xy) + \phi(y) + K$ ③

$\phi(0,0,0) = 0 \therefore K = 0$

Comparison of ① & ③ implies $\phi(z) = 0$

$\phi(xy) = x^2y,\ \phi(yz) = 0,\ \phi(z) = 0,\ \phi(x) = 0$

Comparison of ② & ③ imply $\phi(xz) = 0,\ \phi(y) = 0$

Hence $\phi(x,y,z) = x^2y + xyz$

Indeed this is correct, but not asked for unfortunately

0/2

You needed

$\int F \cdot dr = \phi_1 - \phi_0$
$= \phi_1 - 0$

$\therefore \phi_1 = ab(a+c)$

for a general point

$\phi_1 = xy(x+z)$

iv) $\int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} \underline{F_2} \cdot \underline{dr} = \phi(x_2,y_2,z_2) - \phi(x_1,y_1,z_1)$

(Since F_2 is a conservative vector field, the scalar line integral does not depend on the path taken, only on the end points (x_1,y_1,z_1) and (x_2,y_2,z_2))

$\phi(x_1,y_1,z_1) = \phi(2,1,5) = 2^2\times1 + 2\times1\times5$
$= 14$

$\phi(x_2,y_2,z_2) = \phi(-4,2,-3) = (-4)^2\times2 + (-4)\times2\times(-3)$
$= 16\times2 + 24 = 56$

$\phi(-4,2,-3) - \phi(2,1,5) = 42$

$\therefore \int f_2 \cdot dr = 42$

3/3