$m\ddot{X}_1 = 4kX_2 + \frac{4kl_0}{3} - 4kX_1 - kX_1 - \frac{4kl_0}{3}$

$m\ddot{X}_1 = -5kX_1 + 4kX_2$ as required ✓ 3/3

$m\ddot{x}_2 = k(6l_0 - x_2 - l_0) - 4k(x_2 - x_1 - l_0)$ ✓

$\ddot{X}_2 = \ddot{x}_2$

$m\ddot{X}_2 = k(5l_0 - X_2 - \frac{11l_0}{3}) - 4k(X_2 + \frac{11l_0}{3} - X_1 - \frac{7l_0}{3} - l_0)$

$m\ddot{X}_2 = \frac{4kl_0}{3} - kX_2 - 4kX_2 - \frac{4kl_0}{3} + 4kX_1$

$m\ddot{X}_2 = 4kX_1 - 5kX_2$ as required. ✓ 3/3

iii) Express the system thus

$$\begin{bmatrix}\ddot{X}_1\\ \ddot{X}_2\end{bmatrix} = \begin{bmatrix}-5k/m & 4k/m\\ 4k/m & -5k/m\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}$$

When in normal mode, $\ddot{X}_1 = -\omega^2 X_1$ ✓ ✓

$\ddot{X}_2 = -\omega^2 X_2$

The system becomes

$$\begin{bmatrix}\omega^2 - 5k/m & 4k/m\\ 4k/m & \omega^2 - 5k/m\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$ ① !

2/2

The normal modes eigenvalues are the solution to the determinand of this matrix equated to zero. ✓

ie $\left(\omega^2 - \frac{5k}{m}\right)\left(\omega^2 - \frac{5k}{m}\right) - \left(\frac{4k}{m}\right)^2 = 0$ ✓

$\omega^4 - \frac{10\omega^2 k}{m} + \frac{9k^2}{m^2} = 0.$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\omega^2 = \frac{\frac{10k}{m} \pm \sqrt{\frac{100k^2}{m^2} - 4\times\frac{9k^2}{m^2}}}{2}$

$= \frac{\frac{10k}{m} \pm \sqrt{\frac{64k^2}{m^2}}}{2} = \frac{k}{m}, \frac{9k}{m}$

$\therefore \omega_1 = \sqrt{\frac{k}{m}}$ $\omega_2 = 3\sqrt{\frac{k}{m}}$. ✓ angular frequencies !

2/2

To find the normal mode displacement ratios, find the eigenvectors of matrix ① for ω_1 and ω_2.