Question 3 (*Unit 19*)

Note that the total number of marks available for this question is 14.

Consider the differential equation

$$\frac{dy}{dx} = \sqrt{x} - \sqrt{y}$$

with initial condition $y(1) = 0.5$.

(i) Use the computer package NUMSOL to calculate approximate values for $y(2)$, where y is the solution of the equation with the given initial condition, by Euler's method with step lengths of 0.05, 0.02 and 0.01. From these results, decide whether the use of Euler's method to obtain the value of $y(2)$ accurate to 8 decimal places is sensible. [6]

(ii) Use the computer package NUMSOL to calculate approximate values for $y(2)$, where y is the solution of the equation with the given initial condition, by the Taylor series method of order 2 using three step lengths between 0.01 and 0.2. Find a better estimate for the value of $y(2)$ from these results by extrapolation. Predict how small the step length must be to ensure that the approximation to the value of $y(2)$ obtained using this method is accurate to 8 decimal places. [6]

(iii) Find $y(2)$ to an accuracy of 8 decimal places, explaining how you obtained your result. [2]

Question 4 (*Unit 21*)

Note that the total number of marks available for this question is 10.

This question is concerned with finding the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{bmatrix} 60.14 & -16.70 & -18.30 & 3.32 & -48.60 \\ 154.80 & -37.56 & -62.80 & 25.40 & -131.70 \\ -174.40 & 51.90 & 65.34 & -15.10 & 149.70 \\ -185.70 & 51.60 & 73.30 & -23.46 & 157.10 \\ 75.60 & -28.10 & -20.10 & -4.14 & -61.26 \end{bmatrix}.$$

This problem is stored within the EIGSOL program under the name TMA_2.

(i) Find all the eigenvalues and associated eigenvectors of the matrix $\mathbf{A}$, to an accuracy of 5 decimal places.

You should explain the strategy you adopted for finding all the eigenvalues and eigenvectors, and the reasons for your choice of methods, tolerances, and the values of p and q if you used them. [7]

(ii) The value -18.30 of the element a_{13} in the matrix $\mathbf{A}$ is only approximate. Investigate the absolute conditioning of the eigenvalue of largest modulus with respect to small changes in the value of a_{13}. Explain your strategy. [3]

Question 5 (*Unit 24*)

Note that the total number of marks available for this question is 25.