$\frac{1}{4}$ surely.

$$= \frac{1}{8} + \frac{1}{7} + \frac{1}{3} = \frac{\cancel{21}\ 42 + 24 + 56}{168} = \frac{10\cancel{1}}{168} \quad \frac{122}{168} = \frac{61}{84}$$

×

3/4

iii) $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$
$= at\underline{i} + bt\underline{j} + ct\underline{k}$
$\therefore \frac{d\underline{r}}{dt} = a\underline{i} + b\underline{j} + c\underline{k}$ ✓

and $\underline{F_2} = y(2x+z)\underline{i} + x(x+z)\underline{j} + xy\underline{k}$
$= bt(2at+ct)\underline{i} + at(at+ct)\underline{j} + at \times bt\underline{k}$ ✓
$= bt^2(2a+c)\underline{i} + at^2(a+c)\underline{j} + abt^2\underline{k}$ ✓

3/3

then $\int_{C_2} \underline{F_2} \cdot d\underline{r} = \int_{t_0}^{t_1} \underline{F_2} \cdot \frac{d\underline{r}}{dt} dt$ ✓

becomes $\int_0^1 (bt^2(2a+c)\underline{i} + at^2(a+c)\underline{j} + abt^2\underline{k}) \cdot (a\underline{i} + b\underline{j} + c\underline{k})\, dt$

$= \int_0^1 (abt^2(2a+c) + abt^2(a+c) + abct^2)\, dt$ || common term of $abt^2[\ \ ]$

$= \left[\frac{abt^3(2a+c)}{3} + \frac{abt^3(a+c)}{3} + \frac{abct^3}{3}\right]_0^1$ ✓

$= \frac{ab(2a+c)}{3} + \frac{ab(a+c)}{3} + \frac{abc}{3}$ ✓

$= \frac{ab(2a+c+a+c+c)}{3} = \frac{ab(3a+3c)}{3}$ ✓ || at last – simplification

$= ab(a+c)$ as required. ✓

3/3

iii grad $\phi = \frac{\partial\phi}{\partial x}\underline{i} + \frac{\partial\phi}{\partial y}\underline{j} + \frac{\partial\phi}{\partial z}\underline{k} = \underline{F_2}$

Equating coefficients of $\underline{i}$, $\underline{j}$ and $\underline{k}$ and integrating

|| You were asked to use the result from part(ii)

$\frac{\partial\phi}{\partial x} = y(2x+z)$

$\int \partial\phi = \int y(2x+z)\,dx =$
$\phi(x,y,z) = x^2y + xyz + \phi(y) + \phi(yz) + \phi(z) + K$ ①

usually we write the constant (for x) as $K(y,z)$

$\frac{\partial\phi}{\partial y} = x(x+z)$

$\int \partial\phi = \int x(x+z)\,dy$