Question 5 (*Unit 22*)

This question is concerned with the system of differential equations

$$\dot{x}_1 = 13x_1 + 2x_2 - 20e^{2t},$$
$$\dot{x}_2 = 3x_1 + 8x_2.$$

(i) Express the system of differential equations in the form

$$\dot{\mathbf{x}}(t) = \mathbf{B}\mathbf{x}(t) + \mathbf{h}(t),$$

where

$$\mathbf{x}(t) = [x_1(t) \quad x_2(t)]^T, \qquad \mathbf{h}(t) = [h_1(t) \quad h_2(t)]^T$$

and $\mathbf{B}$ is a constant 2×2 matrix. [2]

(ii) Find the eigenvalues and corresponding eigenvectors of the matrix $\mathbf{B}$. Hence find a matrix $\mathbf{P}$ such that $\mathbf{P}^{-1}\mathbf{B}\mathbf{P}$ is a diagonal matrix. [8]

(iii) Find the general solution of the system of differential equations. [15]

$\dot{x}_1 = 4e^{2t} + 7Ce^{7t} + 28De^{14t} - 26e^{2t} + 7Ce^{7t} + 26De^{14t}$
$+ 2De^{14t} - 2e^{2t} - 6Ce^{7t} - 2De^{14t}$

$-2e^{2t} - 21Ce^{7t} + 14De^{14t} = 6e^{2t} + 3Ce^{7t} + 6De^{14t}$
$+ 8De^{14t} - 3e^{2t} - 24Ce^{7t}$

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