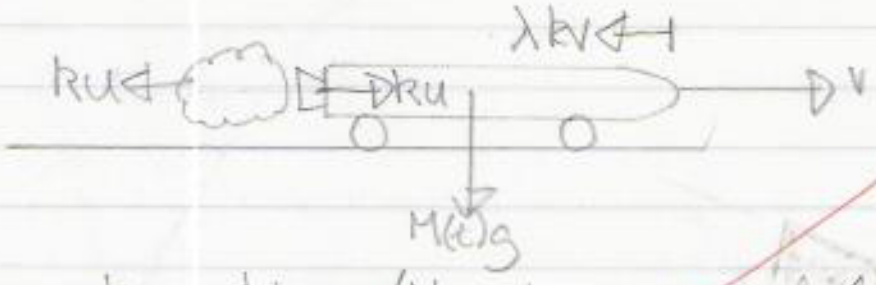
MST 204 05

1)

How is $m(t) = (M - kt)$

Starting from the rector rocket equation (as asked in the question)

derive this



i) $M(t)\frac{dv}{dt} = +ku - \lambda kv$ (Newtons second law of motion)

$(M - kt)\frac{dv}{dt} - ku = -\lambda kv$. This is the one dimensional equation 2/6.

ii) $(M - kt)\frac{dv}{dt} = k(u - \lambda v)$

$\frac{1}{u - \lambda v}dv = \frac{k}{M - kt}dt$ ✓

$\frac{-1}{\lambda}\int_0^v \frac{-\lambda}{u - \lambda v}dv = -\int_0^t \frac{-k}{M - kt}dt$ ✓

$\left[\frac{-1}{\lambda}\ln(u - \lambda v)\right]_0^v = \left[-\ln(M - kt)\right]_0^t$ 6/6

$\frac{-1}{\lambda}\ln(u - \lambda v) + \frac{1}{\lambda}\ln u = -\ln(M - kt) + \ln M$ ✓ 2/2

$\frac{1}{\lambda}\ln\left(\frac{u}{u - \lambda v}\right) = \ln\left(\frac{M}{M - kt}\right)$ ✓

$\ln\left(\frac{u}{u - \lambda v}\right) = \lambda\ln\left(\frac{M}{M - kt}\right) = \ln\left(\left(\frac{M}{M - kt}\right)^{\lambda}\right)$ ✓

$\frac{u}{u - \lambda v} = \left(\frac{M}{M - kt}\right)^{\lambda}$ ✓

$u - \lambda v = u\left(\frac{M - kt}{M}\right)^{\lambda}$ ✓

$v = \frac{u}{\lambda}\left(1 - \left(\frac{M - kt}{M}\right)^{\lambda}\right)$ ✓ 4/4