

iii) $\lambda = 1$, $M - kT = M \cdot \frac{1}{2}$ so $M = 2kT$ ✓

$v = \frac{u}{\lambda} \left(1 - \left(\frac{M - kT}{M} \right)^\lambda \right)$ ✓

~~$\frac{dx}{dt} = \frac{u}{\lambda} \left(1 - \left(\frac{M - kT}{M} \right)^\lambda \right) = u \left(1 - \left(\frac{M - kT}{M} \right) \right) = \frac{ukT}{M}$~~ ✓
3/3

$\int dx = \frac{ukT}{M} dt$

$\int_a^x dx = \int_0^T \frac{ukT}{M} dt$ ✓

$\begin{bmatrix} x \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{ukT^2}{2M} \end{bmatrix}$? 2/2

$x = \frac{ukT^2}{2M} = \frac{ukT^2}{2(2kT)} = \frac{uT}{4}$ ✓ when fuel has all burnt 2/2

2) $f(x) = \cos x$

at $x=0$, $f(x) = \cos 0 = 1$ ✓

at $x=\pi/6$, $f(x) = \cos \pi/6 = \sqrt{3}/2$ ✓

at $x=\pi/3$, $f(x) = \cos \pi/3 = 1/2$ ✓

Put $f(x) = p(x) = a + bx + cx^2$

at $x=0$, $p(x) = a + b(0) + c(0)^2 = 1$ ✓

at $x=\pi/6$, $p(x) = a + b(\pi/6) + c(\pi/6)^2 = \sqrt{3}/2$ ✓

at $x=\pi/3$, $p(x) = a + b(\pi/3) + c(\pi/3)^2 = 1/2$ ✓ 2/2

Put into augmented matrix form and row reduce

$\begin{bmatrix} a & 0 & 0 & 1 \\ a & \frac{b\pi}{6} & \frac{c\pi^2}{36} & \frac{\sqrt{3}}{2} \\ a & \frac{b\pi}{3} & \frac{c\pi^2}{9} & \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} a & 0 & 0 & 1 \\ 0 & \frac{b\pi}{6} & \frac{c\pi^2}{36} & \frac{\sqrt{3}}{2} - 1 \\ 0 & \frac{b\pi}{3} & \frac{c\pi^2}{9} & -\frac{1}{2} \end{bmatrix}$ ✓

Tell me what the row operations are.

$\begin{bmatrix} a & 0 & 0 & 1 \\ 0 & \frac{b\pi}{6} & \frac{c\pi^2}{36} & \frac{\sqrt{3}}{2} - 1 \\ 0 & 0 & \frac{c\pi^2}{18} & \frac{3}{2} - \sqrt{3} \end{bmatrix} \Rightarrow \begin{bmatrix} a & 0 & 0 & 1 \\ 0 & \frac{b\pi}{6} & 0 & \sqrt{3} - \frac{7}{4} \\ 0 & 0 & \frac{c\pi^2}{18} & \frac{3}{2} - \sqrt{3} \end{bmatrix}$

Division signs!

$\begin{bmatrix} a & 0 & 0 & 1 \\ 0 & b & 0 & \frac{\pi}{6}(\sqrt{3} - \frac{7}{4}) \\ 0 & 0 & c & \frac{\pi^2}{18}(\frac{3}{2} - \sqrt{3}) \end{bmatrix} \Rightarrow$

$a = 1$
 $b = -0.03428 \approx -0.0343$ ✓
 $c = -0.4232 \approx -0.4232$ ✓

Correct to four d.p.

$\therefore p(x) = \underline{\hspace{2cm}}$? ✓ 2/3