$x_1 = 0.5640 - \frac{(3\times 0.5640 - 2\times\cos(0.5640))}{(3 + 2\times\sin(0.5640))}$

$= 0.5635692$ ✓

$x_2 = 0.5635692 - \frac{(3\times 0.5635692 - 2\cos(0.5635692))}{(3 + 2\times\sin(0.5635692))}$

$= 0.5635692$ ✓

The value of $|x_2 - x_1| < 10^{-7}$, hence to 6 decimal places. The solution is $x = 0.563569$ ✓ 3/3

b) $\frac{dy}{dx} = \sqrt{x} - \sqrt{y}$

The Taylor series method order 2 gives the recurrence relation

$y_{r+1} = y_r + hy'_r + \frac{h^2}{2}y''_r$ ✓

$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}(\sqrt{x} - \sqrt{y})$ ✓

$= \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{y}}\frac{dy}{dx} = \frac{1}{2\sqrt{x}} - \frac{(\sqrt{x} - \sqrt{y})}{2\sqrt{y}}$ ✓

so $y''_r = \frac{1}{2\sqrt{x_r}} - \frac{(\sqrt{x_r} - \sqrt{y_r})}{2\sqrt{y_r}}$ ✓

The recurrence relation becomes

$y_{r+1} = y_r + h(\sqrt{x_r} - \sqrt{y_r}) + \frac{h^2}{2}\left(\frac{1}{2\sqrt{x_r}} - \frac{(\sqrt{x_r} - \sqrt{y_r})}{2\sqrt{y_r}}\right)$ ✓

State the point you start from $x_0 = 1$, $y_0 = 0.5$, $h = 0.1$

$y(1.1) = 0.5 + 0.1(\sqrt{1} - \sqrt{0.5}) + \frac{0.1^2}{2}\left(\frac{1}{2\sqrt{1.0}} - \frac{(\sqrt{1.0} - \sqrt{0.5})}{2\sqrt{0.5}}\right)$!

$= 0.5 + 0.0341421 - 0.0011756$

$= 0.5353458$ error

$y(1.2) = 0.5353458 + 0.1(\sqrt{1.1} - \sqrt{0.5353458}) + \frac{0.1^2}{2}\left(\frac{1}{2\sqrt{1.1}} - \frac{(\sqrt{1.1} - \sqrt{0.5353458})}{2\times\sqrt{0.5353458}}\right)$ ditto.

$= 0.5353458 + 0.0363772 + 0.0010392$

$= 0.5727622 = 0.572762$ correct to 6 decimal places 4/6

incorrect use of x_0, x_1, x_2

It is often easier to put into table form. Refer to P.25 UNIT 18