multiply through by $\underline{P}^{-1}$ Notation

then $\underline{\dot{y}} = P^{-1}BP\underline{y} + \underline{P}^{-1}\underline{h}(t)$

ie $\begin{pmatrix}\dot{y}_1\\ \dot{y}_2\end{pmatrix} = \begin{pmatrix}7 & 0\\ 0 & 14\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix} + \begin{pmatrix}1/7 & -2/7\\ 3/7 & 1/7\end{pmatrix}\begin{pmatrix}-20e^{2t}\\ 0\end{pmatrix}$ 2/2

(1) $\dot{y}_1 = 7y_1 - 20/7\, e^{2t}$

(2) $\dot{y}_2 = 14y_2 - 60/7\, e^{2t}$

Solve by Integrating factor method. 2/2

For (1) $\dot{y}_1 = 7y_1 - \frac{20}{7}e^{2t}$

$\dot{y}_1 - 7y_1 = -\frac{20}{7}e^{2t}$

Integrating factor is $\exp\int -7\,dt = e^{-7t}$ 2/2

then $\frac{d}{dt}(y_1 e^{-7t}) = -\frac{20}{7}e^{-5t}$

$\int \frac{d}{dt}(y_1 e^{-7t})\,dt = \int -\frac{20}{7}e^{-5t}\,dt$

$y_1 e^{-7t} = \frac{4}{7}e^{-5t} + C$

$y_1 = \frac{4}{7}e^{2t} + Ce^{7t}$ 2/2

For (2) $\dot{y}_2 = 14y_2 - \frac{60}{7}e^{2t}$

$\dot{y}_2 - 14y_2 = -\frac{60}{7}e^{2t}$

Integrating factor is $\exp\int -14\,dt = e^{-14t}$

then $\frac{d}{dt}(y_2 e^{-14t}) = -\frac{60}{7}e^{-12t}$

$\int \frac{d}{dt}(y_2 e^{-14t})\,dt = \int -\frac{60}{7}e^{-12t}\,dt$

$y_2 e^{-14t} = \frac{5}{7}e^{-12t} + D$

$y_2 = \frac{5e^{2t}}{7} + De^{14t}$ 2/2

But $\underline{x} = \underline{P}\underline{y}$