(iii) Use the Newton–Raphson method to determine the root of the equation $2\cos x = 3x$ to an accuracy of 6 decimal places. [6]

(b) Consider the differential equation

$$\frac{dy}{dx} = \sqrt{x} - \sqrt{y} \qquad (x \geq 0)$$

with initial condition $y(1) = 0.5$. Use the Taylor series method of order 2 with step size $h = 0.1$ to find an approximation to $y(1.2)$, where y is the solution to the equation with the given initial condition. Give your answer to six decimal places. [6]

Question 3 (*Unit 19*)

Note that the total number of marks available for this question is 13.

This question is concerned, once again, with the solution of the differential equation

$$\frac{dy}{dx} = \sqrt{x} - \sqrt{y}$$

with the initial condition $y(1) = 0.5$.

(i) Starting from the general form of the recurrence relations for the predictor-corrector (Euler-trapezoidal) method, write down the recurrence relations for finding a numerical solution to the above differential equation using this method. Use your calculator to find an approximation to $y(1.2)$, where y is the solution satisfying the given initial condition, using a step size $h = 0.1$. Give your answer to six decimal places. [5]

(ii) The values shown in the table below were obtained for the approximations to $y(2)$ given by the predictor-corrector (Euler-trapezoidal) method with various step lengths h, by using a computer. Plot a graph of these approximations to $y(2)$ against h^2.

Step length h	Approximation to $y(2)$
0.01	0.895 212 72
0.02	0.895 215 33
0.025	0.895 217 29
0.04	0.895 225 84
0.05	0.895 233 78

What does your graph indicate to be the relationship between the predictor-corrector (Euler-trapezoidal) approximations to $y(2)$ and the step length h for small h? Use your graph to estimate the true value of $y(2)$ to seven decimal places. [4]

(iii) By using one of the entries in the above table and the estimate for the true value of $y(2)$ you obtained in part (ii), calculate the maximum value of the step length h which you would have to use in the predictor-corrector (Euler-trapezoidal) method to ensure an accuracy of eight decimal places in the approximation to $y(2)$. [4]

Question 4 (*Unit 21*)

Note that the total number of marks available for this question is 12.

Find the eigenvalues and corresponding eigenvectors for the matrix

$$\begin{bmatrix} 0 & 3 & 0 \\ -2 & 8 & -2 \\ 1 & 2 & 1 \end{bmatrix}. \qquad [12]$$