The graph implies a positive linear relationship between the square of the step length h, and the corresponding approximation to y(2). The true value of y(2) is estimated from the intersection of the line with the Approximation to y(2) axis (ie $h^2=0$). Reading this off the graph implies a true value for y(2) of 0.8952120 (to seven decimal places) 1/1

iii) $Y_T(1.2) - Y_A(1.2) \approx Ch^2$ 1/1

$0.8952120 - 0.89523378 \approx C \times 0.05^2$

$C = -8.712E-3$ 1/1

|| Use the more accurate value of h = 0.01

To be correct to 8 decimal places,

$|Y_T(1.2) - Y_A(1.2)| \leq 0.5\times10^{-8}$

ie $|Ch^2| \leq 0.5\times10^{-8}$

$h \leq \dfrac{\sqrt{0.5\times10^{-8}}}{\sqrt{|8.712E-3|}} \approx 7.5E-4$

The maximum step length which can be used to ensure accuracy to eight decimal places is thus 7.5×10^{-4} (5×10^{-4} if h is to divide 1 exactly). 2/2

4) $\begin{bmatrix} 0 & 3 & 0 \\ -2 & 8 & -2 \\ 1 & 2 & 1 \end{bmatrix} (= \underline{A})$

eigenvalues are soln to

$\det(\underline{A} - \lambda\underline{I}) = 0$

ie $\det\begin{vmatrix} 0-\lambda & 3 & 0 \\ -2 & 8-\lambda & -2 \\ 1 & 2 & 1-\lambda \end{vmatrix} = 0$

Expand across top row.

$-\lambda\begin{vmatrix} 8-\lambda & -2 \\ 2 & 1-\lambda \end{vmatrix} - 3\begin{vmatrix} -2 & -2 \\ 1 & 1-\lambda \end{vmatrix} = 0$

$-\lambda((8-\lambda)(1-\lambda)+4) - 3(-2(1-\lambda)+2) = 0$

$-\lambda(8-9\lambda+\lambda^2+4) - 3(2\lambda) = 0$