$-\lambda^3 + 9\lambda^2 - 12\lambda - 6\lambda = 0$

$\lambda^3 - 9\lambda^2 + 18\lambda = \lambda(\lambda^2 - 9\lambda + 18) = \lambda(\lambda-6)(\lambda-3) = 0$ ✓

so $\lambda = 0, 3, 6$

4/4

A check here is that
$\sum \lambda_i = \sum a_{ii}$
$\sum \lambda_i = 0+3+6 = ?$
$\sum a_{ii} = 0+8+1 = ?$

To find the eigenvectors

For $\lambda = 0$ $\begin{bmatrix} 0 & 3 & 0 \\ -2 & 8 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

1/2

Row reduce the above matrix

$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 8 & -2 \\ 0 & 3 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 1 \\ -2 & 0 & -2 \\ 0 & 3 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ !

not too clear of the steps that you have used. The result is correct

$\equiv \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ then $x_1 + x_3 = 0$, $x_2 = 0$ ✓

$x_1 + x_3 = 0$ gives the eigenvector $(1, 0, -1)^T$ 2/2 ✓

$\lambda = 3$ $\begin{bmatrix} 0-3 & 3 & 0 \\ -2 & 8-3 & -2 \\ 1 & 2 & 1-3 \end{bmatrix} \equiv \begin{bmatrix} -3 & 3 & 0 \\ -2 & 5 & -2 \\ 1 & 2 & -2 \end{bmatrix}$ ✓

Row reduce the matrix

$\begin{bmatrix} -3 & 3 & 0 \\ -2 & 5 & -2 \\ 1 & 2 & -2 \end{bmatrix} \equiv \begin{bmatrix} 1 & -1 & 0 \\ -2 & 5 & -2 \\ 1 & 2 & -2 \end{bmatrix} \equiv \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{bmatrix}$

$\equiv \begin{bmatrix} 1 & 0 & -2/3 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & -2/3 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & -2/3 \\ 0 & 1 & -2/3 \\ 0 & 0 & 0 \end{bmatrix}$

? There is no need for this. From above;
$-3x_1 + 3x_2 = 0$
If $x_1 = k$, $x_2 = k$.
Then $-2x_1 + 5x_2 - 2x_3 = 0$
hence $x_3$

then $x_1 - \frac{2}{3}x_3 = 0$, $x_2 - \frac{2}{3}x_3 = 0$

Put $x_3 = k$, then $x_1 = 2k/3$ and $x_2 = 2k/3$

An eigenvector is then $(2k/3, 2k/3, k)^T$ ✓ 2/2

or $(2, 2, 3)^T$

$\lambda = 6$ $\begin{bmatrix} 0-6 & 3 & 0 \\ -2 & 8-6 & -2 \\ 1 & 2 & 1-6 \end{bmatrix} \equiv \begin{bmatrix} -6 & 3 & 0 \\ -2 & 2 & -2 \\ 1 & 2 & -5 \end{bmatrix}$