This assignment covers *Units 17, 18, 19, 21* and *22*.

There are five questions in this assignment; Question 3 and Question 4 are both allotted fewer than 25 marks, however, and the total number of marks for the assignment is 100.

Note that there is no tutor-marked assignment question on *Unit 20*; this unit is assessed by computer-marked assignment questions only.

Question 1 (*Unit 17*)

A test rig for a rocket motor consists of a trolley on which the rocket is rigidly mounted and which runs on a straight horizontal railway track. The rocket is positioned so that its exhaust gases are ejected horizontally and parallel to the railway track. The rocket burns fuel at a constant rate k, and the exhaust gases are ejected horizontally at a constant speed u relative to the rocket. The total initial mass of the trolley, the rocket and its fuel is M. During the motion, the trolley experiences a resistive force (due to friction and air resistance) whose magnitude is proportional to the trolley's speed. The trolley is initially at rest; the rocket motor is then started.

(i) Starting from the (vector) rocket equation, show that after the rocket motor has been running for a time t, the speed v of the trolley satisfies the differential equation

$$(M - kt)\frac{dv}{dt} - ku = -\lambda kv,$$

where for convenience the magnitude of the resistive force has been expressed as λkv. [6]

(ii) Show that the speed of the trolley at time t (supposing that the fuel is not yet exhausted) is given by

$$v = \frac{u}{\lambda}\left(1 - \left(\frac{M - kt}{M}\right)^{\lambda}\right).$$ [12]

(iii) In a particular experiment, $\lambda = 1$, half of the total initial mass is made up of fuel, and the fuel lasts for time T. Find an expression (in terms of u and T) for the distance travelled by the trolley from its starting point to the point at which it runs out of fuel. [7]

Question 2 (*Unit 18*)

(a) (i) Show that the quadratic polynomial $p(x)$ which interpolates the function

$$f(x) = \cos x$$

at the points $x = 0$, $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ is

$$p(x) = 1 - 0.0343x - 0.4232x^2,$$

where the coefficients are correct to 4 decimal places.

On the same set of axes, sketch the graphs of $f(x)$ and $p(x)$ over the interval $\left[-\frac{1}{2}\pi, \pi\right]$, and comment on your sketches. [8]

(ii) The equation

$$2\cos x = 3x$$

has a single root, which is near $\frac{1}{6}\pi$. By using $p(x)$ as an approximation to $\cos x$, find an approximation to this root better than $\frac{1}{6}\pi$, giving your answer to 4 decimal places. [5]