3) i) $y^*_{r+1} = y_r + hy'_r$ where $y'_r = \sqrt{x_r} - \sqrt{y_r}$

ie $y^*_{r+1} = y_r + h(\sqrt{x_r} - \sqrt{y_r})$

and $y_{r+1} = y_r + \frac{1}{2}h(y'_r + y^{*\prime}_{r+1})$ 1/1

where $y^{*\prime}_{r+1} = \sqrt{x_{r+1}} - \sqrt{y^*_{r+1}}$

so $y_{r+1} = y_r + \frac{1}{2}h(\sqrt{x_r} - \sqrt{y_{r}} + \sqrt{x_{r+1}} - \sqrt{y^*_{r+1}})$

The recurrence relations are

$$y^*_{r+1} = y_r + h(\sqrt{x_r} - \sqrt{y_r})$$

and $y_{r+1} = y_r + \frac{h}{2}(\sqrt{x_r} - \sqrt{y_r} + \sqrt{x_{r+1}} - \sqrt{y^*_{r+1}})$

To find $y(1.1)$

$y^*_1 = y_0 + h(\sqrt{x_0} - \sqrt{y_0})$

$= 0.5 + 0.1(\sqrt{1} - \sqrt{0.5})$

$= 0.5292893$

$y_1 = y_0 + \frac{h}{2}(\sqrt{x_0} - \sqrt{y_0} + \sqrt{x_1} - \sqrt{y^*_1})$

$= 0.5 + 0.05(\sqrt{1} - \sqrt{0.5} + \sqrt{1.1} - \sqrt{0.5292893})$

$= 0.530709$ keep to 7 d.p. until the final answer

To find $y(1.2)$

$y^*_2 = y_1 + h(\sqrt{x_1} - \sqrt{y_1})$

$= 0.530709 + 0.1(\sqrt{1.1} - \sqrt{0.530709})$

$= 0.5627401$

$y_2 = y_1 + \frac{h}{2}(\sqrt{x_1} - \sqrt{y_1} + \sqrt{x_2} - \sqrt{y^*_2})$

$= 0.530709 + 0.05(\sqrt{1.1} - \sqrt{0.530709} + \sqrt{1.2} - \sqrt{0.5627401})$

$= 0.563989$ (to six sig figures)

$3\frac{1}{2}/4$