Row reduce the matrix

$$\begin{bmatrix} -6 & 3 & 0 \\ -2 & 2 & -2 \\ 1 & 2 & -5 \end{bmatrix} \equiv \begin{bmatrix} 2 & -1 & 0 \\ -2 & 2 & -2 \\ 1 & 2 & -5 \end{bmatrix} \equiv \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -2 \\ 1 & 2 & -5 \end{bmatrix}$$

again no need for this.

$$\equiv \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -2 \\ 0 & 5/2 & -5 \end{bmatrix} \equiv \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 5/2 & -5 \end{bmatrix} \equiv \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

then $x_1 - x_3 = 0 \quad x_2 - 2x_3 = 0$

An eigenvector is then $(1, 2, 1)^T$ ✓ 2/2

5) i) $\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 13 & 2 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} -20e^{2t} \\ 0 \end{pmatrix}$ ✓ 2/2

ii) To find eigenvalues

$\det \begin{pmatrix} 13-\lambda & 2 \\ 3 & 8-\lambda \end{pmatrix} = 0$ ✓

$(13-\lambda)(8-\lambda) - 6 = 104 - 21\lambda + \lambda^2 - 6 = 0$

$\lambda^2 - 21\lambda + 98 = 0$

$(\lambda - 14)(\lambda - 7) = 0 \Rightarrow \lambda = 7, 14.$ ✓ 3/3

Check
$\Sigma \lambda_i =$?
$\Sigma a_{ii} =$?

$\underline{\lambda = 7}$ $\begin{pmatrix} 13-7 & 2 \\ 3 & 8-7 \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 3 & 1 \end{pmatrix}$

eigenvector is soln to $3x_1 + x_2 = 0$ ie $(1, -3)^T$ ✓ 2/2

$\underline{\lambda = 14}$ $\begin{pmatrix} 13-14 & 2 \\ 3 & 8-14 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 3 & -6 \end{pmatrix}$

eigenvector is soln to $-x_1 + 2x_2 = 0$ ie $(2, 1)^T$ ✓ 2/2

Hence $\underline{P} = \begin{pmatrix} 1 & 2 \\ -3 & 1 \end{pmatrix}$ ✓ 1/1

$\underline{P}^{-1} = \frac{1}{(1\times1 - 2\times-3)} \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1/7 & -2/7 \\ 3/7 & 1/7 \end{pmatrix}$ ✓ 2/2

$\underline{P}^{-1}\underline{B}\underline{P} = \begin{pmatrix} 7 & 0 \\ 0 & 14 \end{pmatrix}$ ✓ 1/1

iii) Sub $\underline{x}(t) = \underline{P}\underline{y}(t)$ Vector Notation !

then system becomes

$\underline{P}\dot{\underline{y}}(t) = \underline{B}\underline{P}\underline{y}(t) + \underline{h}(t)$