Question 2

In this question you are asked to carry out some mathematical modelling for yourself. We shall first describe the situation, then make some suggestions about how you should tackle the problem, then give you some data, and finally explain what your tutor will be looking for in your solution.

The situation

There was a report recently on the local TV news for the East Anglia region about a campaign being mounted by the residents of a village in Bedfordshire called Millbrook, to persuade Bedfordshire County Council to install speed humps along the main road through the village. It appears that motorists frequently ignore the 30 mph speed limit as they drive through the village. Fences and walls are regularly knocked down when cars go out of control. Fortunately, no one has been hurt in an accident yet, but pedestrians, especially children, are often in danger from speeding traffic.

It would improve the case which the people of Millbrook intend to present to Bedfordshire County Council if they could specify how many speed humps are needed, and how far apart they should be spaced. You are invited to use your mathematical modelling skills to solve this problem.

Method of attack

First, you must be sure you understand how speed humps are intended to produce the effect of reducing traffic speed. A speed hump, or 'sleeping policeman', is a bump or similar form of surface irregularity stretching the full width of the road, which can be crossed in comfort (and without the risk of damaging the car) only at quite low speeds. A succession of speed humps controls the speed of traffic along the stretch of road where they are installed, because there is not sufficient time for a motorist to get up speed after crossing one before it is necessary to brake on approaching the next. A car travelling on roads with speed humps usually proceeds as follows. Immediately after having negotiated one hump, the car accelerates, with constant acceleration, until it becomes necessary to brake for the next. While the brakes are operating, the car's deceleration is constant. The brakes are applied continuously until the car reaches the next hump, at which point its speed is sufficiently small that the hump can be crossed safely. The car attains its maximum speed at the instant the brakes are applied to begin the deceleration for the next hump. All other things being equal, the farther apart the humps are, the greater will be the maximum speed a car can attain between them, and the closer they are together, the smaller that maximum speed will be.

In order to determine how far apart to space the humps, therefore, the first task is to work out how the distance between the humps determines a car's maximum speed. It will also depend on other things, of course, such as the speed of the car as it goes over the hump, and its acceleration and deceleration – but these can be treated as parameters. You should model the motion of a car between two successive humps as a period of constant acceleration followed by a period of constant deceleration, and you should aim to derive an equation which relates the distance between the humps to the maximum speed attained between them, together with such other parameters as are required. Using this equation, and the data given below, you should work out how far apart the humps should be to keep traffic speed below a desirable speed limit. Note that you are asked only to find how far apart the humps are to be spaced. You are not asked to model the up and down motion of a vehicle when crossing a hump.

Data

Following is a sketch map of Millbrook.