From equation (7) we see that with $y = 0$ the time taken for the bath to empty is

$$t_h = \frac{2\sqrt{h}}{k}, \tag{8}$$

and with $y = h/2$ the time taken for the first half of the bath to empty is

$$t_{h/2} = \boxed{\text{Deliberate omission!}} \tag{9}$$

Combining (8) and (9), we obtain the time taken for the last half of the bath to empty as

$$t_h - t_{h/2} = \boxed{\text{Deliberate omission!}} \tag{10}$$

Thus the answer to the question "how much longer does the last half of the bath take to empty compared with the first half?" is found from (9) and (10) as

$$\frac{t_h - t_{h/2}}{t_{h/2}} = \sqrt{2} + 1. \tag{11}$$

Therefore, the last half takes approximately 2.4 times as long to empty as the first half. Furthermore, the first half and second half take approximately 30 per cent and 70 per cent, respectively, of the total time taken. This is a consequence of the principle that "pressure increases with depth". Now, although these results are perhaps a little surprising, they are easily verified with a few experiments, and I suggest readers try it for themselves!

The passage above is taken from an article by P. Glaister which was published in the journal 'Teaching Mathematics and its Applications', Volume 9, No. 1, 1990. We have made some minor modifications for the purposes of the question.

(i) *Specify the real problem*

Clearly, the article is concerned with what happens to the bath water after the plug is pulled out; but this does not state the problem sufficiently precisely for the purposes of mathematical modelling. In fact, the author asks two questions: a quite specific one, which is answered at the end of the passage, and another more general one, which in effect is answered in Equation (7). Identify both of these. You should be able to specify each in one sentence. [4]

(ii) *Set up a model*

(a) There are many symbols in the passage: $A, h, S, y, \ldots$. Find them all, explain precisely what each represents ('t is time' will *not* do!), and give appropriate units of measurement. [5]

(b) The mathematical model – which is the differential equation given in Equation (6) – is derived in several stages, starting with Equation (1). State the principle (which you met in *Unit 3*) on which the derivation of Equation (1) is based. (Do *not* reproduce the derivation in the paragraph leading up to Equation (1).) [1]

Explain why $V = Ay$. (A correct answer will describe why it is not actually necessary for the bath to be rectangular for this formula to hold.) Hence show how Equation (3) follows from Equation (2), and explain how Equation (6) follows from Equation (5). [5]

(c) As the author says at the end of the first paragraph, a number of 'ideal assumptions [are] made in order to set up the model'. However, authors of articles on modelling sometimes overlook assumptions that in fact are used implicitly; and they sometimes state assumptions that are not really necessary. Each of the following statements could possibly be an assumption needed to develop this model. For each statement, decide first, whether or not it (or something equivalent) is explicitly mentioned in the passage, and secondly, whether or not it is actually necessary to make that assumption in order to derive the model in the way described in the passage.