- The water in the bath can be treated as a collection of particles which obey the law of conservation of energy.
- The plug hole is circular.
- The cross-sectional area of the bath is the same at every height.
- The taps are turned off throughout.

Find one more necessary assumption, not mentioned in the list above. Use your own words for your answer – do not merely copy from the passage. [5]

(iii) *Formulate the mathematical problem*

In part (ii)(b) above, we said that the mathematical model is the differential equation given in Equation (6). Which of the following would you regard as the 'mathematical problem' in this article?

- To solve Equation (4) for $v(t)$.
- To calculate the value of k from the formula given after Equation (6).
- To find the solution of the differential equation with the initial condition given in Equation (6).
- To calculate the value of $\sqrt{2}+1$ correct to one decimal place.

Or is it something else entirely? [2]

(iv) *Solve the mathematical problem*

The steps leading from Equation (6) to Equation (7) have been omitted from the version of the passage printed above. Reconstruct them. (Merely showing that Equation (7) gives a solution, by substituting it into Equation (6), will not receive any marks.) [8]

(v) *Interpret the solution*

(a) One qualitative check of the solution is to sketch a graph of y against t, to confirm that the solution has the features one would expect from ordinary observation. The author does include such a graph, in Figure 2 – but we have omitted the actual curve. Complete Figure 2, and describe what features of your graph correspond to the expected behaviour of emptying baths. [4]

(b) The passage concludes with the following statements: '... the last half takes approximately 2.4 times as long to empty as the first half. Furthermore, the first half and second half take approximately 30 per cent and 70 per cent, respectively, of the total time taken.' Explain how these results are deduced from Equation (7). Your answer should begin by explaining how Equation (8) is derived, and should include the expressions which have been deliberately omitted from Equations (9) and (10). [6]

(vi) *Compare with reality*

(a) Here are some data obtained by carrying out measurements with a domestic bath of standard shape. The measurements were carried out with a ruler to measure the depth of water above the plug hole, and a digital clock to record the time. The bath was filled to a depth of 8 inches, and took 1 minute and 46 seconds to empty. The time taken for the water level to drop to 4 inches was 42 seconds. Compare these observations with the predictions of the model by calculating $(t_h - t_{h/2})/t_{h/2}$ for these data. If there is any disagreement, suggest why it may have arisen when using a domestic bath for validation purposes. [5]

(b) In the light of part (vi)(a), say which one of the assumptions would need to be revised or relaxed so that the model could be improved in order to deal more realistically with the emptying of a domestic bath. Describe how the assumption should be changed, and explain briefly what effects the change will have on the derivation of Equation (6). [5]