

I have stopped putting in the vector notation perhaps you might complete for practice

$$= 2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$$

$$\vec{PQ} \text{ increases at rate } \vec{v}_Q - \vec{v}_P = (-\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}) - (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$$

$$\text{ie } \dot{\vec{PQ}} = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$$

$$\int_{2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}}^{\vec{PQ}} \dot{\vec{PQ}} = \int_0^t (-2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) dt \quad \checkmark$$

$$[\vec{PQ}]_{2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}}^{\vec{PQ}} = [-2t\mathbf{i} + 3t\mathbf{j} - 4t\mathbf{k}]_0^t \quad \checkmark$$

$$\vec{PQ} - (2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) = -2t\mathbf{i} + 3t\mathbf{j} - 4t\mathbf{k}$$

$$\vec{PQ} = (2 - 2t)\mathbf{i} + (-4 + 3t)\mathbf{j} + (6 - 4t)\mathbf{k}$$

$$\text{Initially } \vec{PR} = \vec{r}_R - \vec{r}_P = (\mathbf{i} + \mathbf{j} + \mathbf{k}) - (-\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$$

$$\vec{PR} \text{ increases at rate } \vec{v}_R - \vec{v}_P = (2\mathbf{j} + \mathbf{k}) - (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = -\mathbf{i} + \mathbf{j} - \mathbf{k} \quad \checkmark$$

$$\text{ie } \dot{\vec{PR}} = -\mathbf{i} + \mathbf{j} - \mathbf{k}$$

$$\int_{2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}}^{\vec{PR}} \dot{\vec{PR}} = \int_0^t (-\mathbf{i} + \mathbf{j} - \mathbf{k}) dt$$

$$[\vec{PR}]_{2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}}^{\vec{PR}} = [-t\mathbf{i} + t\mathbf{j} - t\mathbf{k}]_0^t \quad \checkmark$$

$$\vec{PR} - (2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = -t\mathbf{i} + t\mathbf{j} - t\mathbf{k}$$

$$\vec{PR} = (2 - t)\mathbf{i} + (-3 + t)\mathbf{j} + (4 - t)\mathbf{k} \quad \checkmark$$

6) A vector perpendicular to the triangle PQR can be found by taking the cross product of  $\vec{PQ}$  and  $\vec{PR}$ .

$$\begin{aligned} \vec{PQ} \times \vec{PR} &= ((2 - 2t)\mathbf{i} + (-4 + 3t)\mathbf{j} + (6 - 4t)\mathbf{k}) \times ((2 - t)\mathbf{i} + (-3 + t)\mathbf{j} + (4 - t)\mathbf{k}) \\ &= ((-4 + 3t)(4 - t) - (6 - 4t)(-3 + t))\mathbf{i} + ((6 - 4t)(2 - t) - (2 - 2t)(4 - t))\mathbf{j} \\ &\quad + ((2 - 2t)(-3 + t) - (-4 + 3t)(2 - t))\mathbf{k} \\ &= (-16 + 16t - 3t^2 + 18 - 18t + 4t^2)\mathbf{i} + (12 - 14t + 4t^2 - 8 + 10t - 2t^2)\mathbf{j} \\ &\quad + (-6 + 8t - 2t^2 + 8 - 10t + 3t^2)\mathbf{k} \\ &= (2 - 2t + t^2)\mathbf{i} + (4 - 4t + 2t^2)\mathbf{j} + (2 - 2t + t^2)\mathbf{k} \\ &= (2 - 2t + t^2)(\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \quad \checkmark \end{aligned}$$

$2 - 2t + t^2$  is a number, hence the normal to the triangle is always parallel to  $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ .  $\checkmark$

you could have obtained this by using  $\vec{r} = \vec{r}_0 + \vec{v}t$  for  $\vec{r}_Q$ , and  $\vec{r}_R$  and then subtract them.