$\dot{y} = 10\sqrt{5}\sin\alpha - gt$ ✓

$$\int_0^y \dot{y}\,dt = \int_0^t (10\sqrt{5}\sin\alpha - gt)\,dt$$

$y = 10t\sqrt{5}\sin\alpha - \frac{gt^2}{2}$ ✓ 3/4

but $t = \frac{x}{10\sqrt{5}\cos\alpha}$ ✓

so $y = \frac{10x\sqrt{5}\sin\alpha}{10\sqrt{5}\cos\alpha} - \frac{g}{2}\left(\frac{x}{10\sqrt{5}\cos\alpha}\right)^2$

$= x\tan\alpha - \frac{10x^2\sec^2\alpha}{1000}$ ✓

$= x\tan\alpha - \frac{x^2}{100}\sec^2\alpha$ ✓ 3/3

iii) In order for the ball to hit the stone, it must pass through the point (10, 20) ✓

$y = x\tan\alpha - \frac{x^2}{100}\sec^2\alpha$

$20 = 10\tan\alpha - \frac{10^2}{100}(1+\tan^2\alpha)$ ✓

$20 = 10\tan\alpha - 1 - \tan^2\alpha$

$\tan^2\alpha - 10\tan\alpha + 21 = 0$ ✓

$(\tan\alpha - 7)(\tan\alpha - 3) = 0$ ✓

$\tan\alpha = 7$ or 3 $(0 \le \alpha \le \pi/2)$ 4/4

$\tan\alpha = 7$, $\cos\alpha = \frac{1}{\sqrt{1+7^2}} = \frac{1}{5\sqrt{2}}$

$t = \frac{x}{10\sqrt{5}\cos\alpha} = \frac{10}{10\sqrt{5}\times\frac{1}{5\sqrt{2}}}$

$= \sqrt{10}\,s$ ✓

$\tan\alpha = 3$, $\cos\alpha = \frac{1}{\sqrt{1+3^2}} = \frac{1}{\sqrt{10}}$

$t = \frac{x}{10\sqrt{5}\cos\alpha} = \frac{10}{10\sqrt{5}\times\frac{1}{\sqrt{10}}}$

$= \sqrt{2}\,s$ ✓ 3/3.

The girl throws the ball, and after two seconds it reaches it's maximum height. Hence the stone should take less than 2 seconds to reach the same point, so the stone should be thrown $(2-\sqrt{2})$ seconds after the ball is thrown ($= 0.586s$). ✓ 2/2

$\alpha = \tan^{-1}3 = 71.56°$ ✓ 1/1.