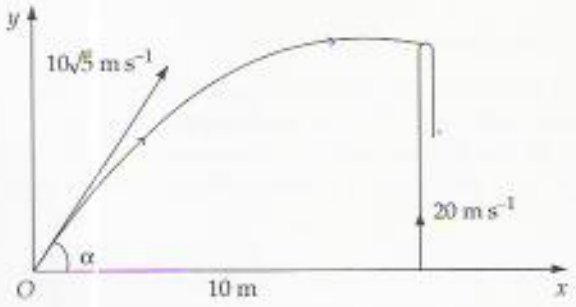
Question 4 (*Unit 15*)



A girl throws a ball vertically upwards with a speed of $20\,\mathrm{m\,s^{-1}}$. A boy standing 10 metres away subsequently throws a stone at the ball. The speed of projection of the stone is $10\sqrt{5}\,\mathrm{m\,s^{-1}}$, and the ball and the stone are thrown from the same level. Throughout the question, you should neglect air resistance and model the ball and stone as particles which experience only the force of gravity during their flights. You should assume that the acceleration due to gravity is constant and of magnitude $g = 10\,\mathrm{m\,s^{-2}}$.

(i) Find the maximum height of the ball in its flight, and how long it takes to reach this height. [5]

(ii) *From Newton's second law of motion*, show that the horizontal and upward vertical displacements x and y of the stone from its point of projection satisfy the equation

$$y = x\tan\alpha - \tfrac{1}{100}x^2\sec^2\alpha,$$

where α is the angle of projection of the stone. [10]

(iii) Find the angle of projection of the stone, and the time at which the boy should throw it, in order that the ball will be hit by the stone when the ball is at the highest point of its trajectory. [10]

[*You may find the following identity useful*:

$$\sec^2\alpha = 1 + \tan^2\alpha.]$$

M = LT
M⁻²