7. The heat required to bring the contents and fabric of any room up to the required temperature, from cold, is insignificant compared with the loss of heat to the outside.
8. When the heating is switched on in a particular zone, the temperature increase from cold to the required temperature occurs instantaneously. Similarly, when the heating is switched off, the temperature drop in the zone occurs instantaneously.
9. Annual interest on capital does not change from year to year.
10. Inflation may be ignored.

The following data may be used in solving the problem.

a. The house is 10 metres deep and 6 metres wide, and each storey is 2.5 metres high.
b. Each floor has a total window area of $8\,\mathrm{m}^2$.
c. Heating is required in the average house in an average year for 200 days during the year, and on those days the average external temperature is 5 °C.
d. Running costs of the heating system are 5 pence per kilowatt-hour. (One kilowatt-hour, or kW h, is the energy produced by a source rated at 1 kW running for one hour.)
e. When the heating is on, a temperature of 20 °C is maintained in the zone.
f. A typical U value for the external walls of a modern house is $0.56\,\mathrm{W\,m^{-2}\,°C^{-1}}$.
g. A typical U value for the windows of a modern house, assumed to be double-glazed, is $2.9\,\mathrm{W\,m^{-2}\,°C^{-1}}$.
h. A typical U value for the upstairs ceiling of a modern house is $0.34\,\mathrm{W\,m^{-2}\,°C^{-1}}$.
i. A typical U value for the downstairs floor of a modern house is $0.46\,\mathrm{W\,m^{-2}\,°C^{-1}}$.
j. A typical U value for the downstairs ceiling and upstairs floor of a modern house is $1.0\,\mathrm{W\,m^{-2}\,°C^{-1}}$.
k. When the heating is on, it is on downstairs from 7 o'clock in the morning to 11 o'clock at night; it is on upstairs from 7 o'clock to 9 o'clock in the morning and from 9 o'clock to 11 o'clock in the evening.
l. The rate of annual interest on capital is 3%.

upstairs 7 8 9 10 11 12 13 14 15 16 17 18 19 20

The following symbols are used in the statement of the question.

- T_u, the upstairs temperature, in °C.
- T_d, the downstairs temperature, in °C.
- T_o, the outside temperature, in °C.
- q_a, the rate of loss of heat, in watts, from the upstairs to the outside through the walls, windows and roof, when the temperature inside is T_u °C and the temperature outside is T_o °C.
- q_b, the rate of flow of heat, in watts, between the downstairs and the upstairs when the temperature upstairs is T_u °C and the temperature downstairs is T_d °C.

upstairs 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
d'stairs 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23

If you use any other symbols in your solution, you should define them carefully.

(i) Give as precise a statement as you can of the problem to be solved, in your own words. [3]

(ii) Show that q_a, the rate of loss of heat, in watts, from the upstairs through the walls, windows and ceiling, is given by

$$q_a = 83.92(T_u - T_o).$$

Derive also a similar expression for q_b in terms of T_u and T_d. [5]

(iii) Find the rate of loss of heat, in watts, from the upstairs through the walls, windows and ceiling, when the heating is on both upstairs and downstairs, the temperature inside is 20 °C and the temperature outside is 5 °C. [1]

22 - 9 = 14
21 - 9 = 12