This assignment covers *Units 9, 12, 14* and *15*.

Question 1 (*Unit 9*)

Use Gaussian elimination methods to determine whether the following sets of equations have

- no solution,
- a unique solution,
- an infinite number of solutions.

If the set of equations has a unique solution find the solution, and if the set of equations has an infinite number of solutions find the general solution.

(a)
$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 4x_1 + x_2 + 3x_3 &= -3 \\ 2x_1 - x_2 + 2x_3 &= -8 \end{aligned}$$
[11]

(b)
$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 4x_1 + x_2 + 3x_3 &= -3 \\ 2x_1 - x_2 + x_3 &= -8 \end{aligned}$$
[5]

(c)
$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 4x_1 + x_2 + 3x_3 &= -3 \\ 2x_1 - x_2 + x_3 &= -7 \end{aligned}$$
[9]

Question 2 (*Unit 12*)

This question is concerned with the financial benefits which may be derived from having zone heating in a house. With central heating, the whole house is heated from a central boiler; there is a single time-clock to control the boiler and pump. This can be wasteful if heat is supplied to rooms which are not used during the day. In the case of zone heating, there is still only one boiler and pump, but the radiators are divided into zones according to their pattern of use, and each zone is controlled independently by its own time-clock. This question deals with a two-zone system, one zone consisting of the living rooms downstairs which are kept warm throughout the day, while the other zone consists of the bedrooms upstairs where the heating is switched on for a brief period in the morning and again in the evening, at the times when these rooms are being used. This system saves the householder money by reducing the cost of heating; but against this saving must be set the cost of installing the controls themselves.

The cost of heating is a continuing, or recurrent, cost whereas expenditure on the extra controls is a one-off, capital, cost. The standard method of comparing a recurrent and a capital cost is to compare the recurrent cost with the interest which would have been received if the capital had been invested instead of spent.

The problem is to find how great the cost of installing zone heating may be while still giving financial benefit to the householder, taking into account both the cost of heating and the expenditure on the extra controls.

The model of the finances of zone heating developed in this question is based on the following assumptions.

1. A two-storey detached dwelling, in which the heating on each floor is controlled separately, is considered.
2. The temperature on each floor is uniform.
3. The external walls of the house are of uniform construction.
4. All the windows of the house are double-glazed.
5. The effect of heat loss through external doors is negligible.
6. The roof space has the same temperature as outside.