c) $x_1 + x_2 + x_3 = 2$ E_1
$4x_1 + x_2 + 3x_3 = -3$ E_2
$2x_1 - x_2 + x_3 = -7$ E_3

Eliminate x_1 from E_2 and E_3

$E_{2a}: E_2 - 4 \times E_1$
$E_{2a}: -3x_2 - x_3 = -11$ ✓

$E_{3a}: E_3 - 2 \times E_1$
$E_{3a}: -3x_2 - x_3 = -11$ ✓ 2/2

Eliminate x_2 from E_{3a}

$E_{3b}: E_{3a} - E_{2a}$
$E_{3b}: 0 = 0$ $0x_1 + 0x_2 + 0x_3 = 0$

Same point 1/2

Hence the system of equations is linearly dependent; in effect we have only two equations, E_1 and E_{2a}. By giving x_3 an arbitrary value, s, we can find an infinite number of solutions. // Most commonly we use k ie let $x_3 = k$.

From $E_{2a}: -3x_2 - x_3 = -11$

$3x_2 = -x_3 + 11 = -s + 11$

$x_2 = 11/3 - s/3$ ✓

Sub for x_3 and x_2 in E_1

$E_1: x_1 + x_2 + x_3 = 2$

so $x_1 = 2 - x_2 - x_3$

$= 2 - (11/3 - s/3) - s$

$= 2 - 11/3 + s/3 - s$

$= -5/3 - 2/3 s$ ✓

The general solution is therefore

$x_1 = -5/3 - 2s/3$ ✓

$x_2 = 11/3 - s/3$ ✓

$x_3 = s$ ✓ 5/5

Please refer to similar solutions in the UNIT 9 (P.19)

2) i) The problem is to use the assumptions of the model to calculate whether the savings in the cost of heating the house will ✓