Question 2 (*Unit 6*)

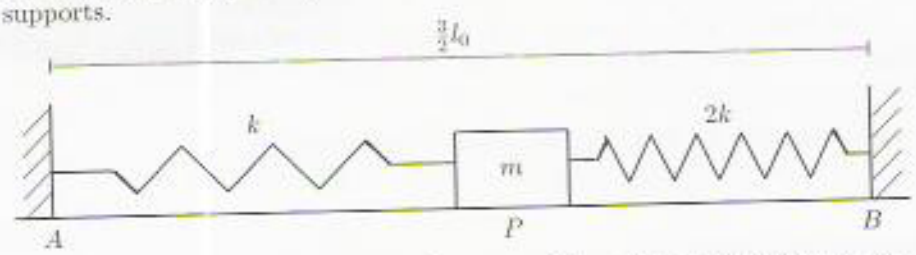
(i) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 25x - 3e^{2x}.$$ [17]

(ii) Hence find the particular solution of the above differential equation which satisfies the initial conditions $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$. [8]

Question 3 (*Unit 7*)

(a) The figure shows a particle P which moves on a frictionless horizontal track. The particle, whose mass is m, is attached by two perfect springs to fixed supports at A and B. The spring AP has stiffness k, while the spring PB has stiffness $2k$. Both springs have natural length l_0. The distance AB between the supports is $\frac{3}{2}l_0$. The particle moves so as to remain always between the supports.



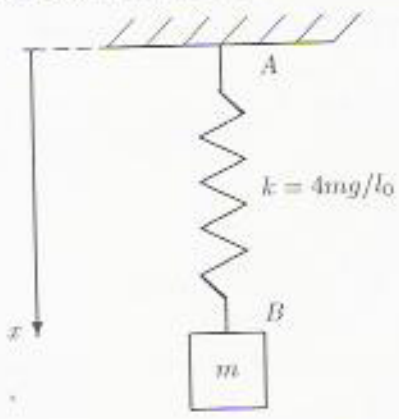
(i) On a similar diagram, representing a possible configuration of the system, draw arrows showing the directions of the forces acting on the particle due to the springs. Explain clearly which spring(s) you have decided to regard as extended and which as compressed. [3]

Take the x-axis to be along the track from A to B, with origin at A.

(ii) Derive the equation of motion of the particle. [6]

(iii) Find the general solution of the differential equation obtained in part (ii). Show that the static position of the particle (the position in which it can remain at rest) is a distance $\frac{3}{3}l_0$ from A. Write down the period of oscillations of the particle when it is in motion between the supports. [6]

(b)



A particle of mass m is attached to one end B of a light perfect spring AB of natural length l_0 and stiffness k. The spring is hung vertically with the end A fixed and B below A. The particle's mass, the stiffness of the spring and the natural length of the spring are related by the equation $kl_0 = 4mg$, where g is the magnitude of the acceleration due to gravity.

(i) Determine an expression for the total mechanical energy of the system when the particle is at a distance x below A. [4]

(ii) Initially, the particle has velocity $\sqrt{(2gl_0)}$ downwards and the spring has its natural length. By using the conservation of energy, find the range of values of x over which the particle moves in its subsequent motion. [6]