This assignment covers *Units 4, 6, 7* and *8*.

Note that there is no tutor-marked assignment question on *Unit 5*; this unit is assessed by computer-marked assignment questions only.

Question 1 (*Unit 4*)

This question is devoted to the development of a model of parachuting. A parachute jump consists of two parts: first, a period of free fall; second, a period of fall with the parachute open. It is assumed that in free fall the only force acting on the parachutist is his weight, but that with the parachute open the parachutist experiences another, resistive, force of magnitude kv^2 newtons where v m s^{-1} is his speed and k is a constant depending on the size and construction of the parachute. One of the aims of the question is to find an appropriate value for k. The parachutist and parachute are to be modelled throughout as a particle.

Consider first the period of free fall.

(i) Supposing that initially the parachutist is at rest, find expressions for his speed

(a) in terms of the time t he has been falling, and

(b) in terms of the distance x he has fallen. [3]

Consider now the period of the parachutist's motion with the parachute open.

(ii) Write down the equation of motion of the parachutist, with open parachute, relative to an axis directed vertically downward with origin at the point where the parachute is opened. State clearly the meanings of the symbols you use. [4]

(iii) Find an expression for v_T, the terminal speed of the parachutist, in terms of his mass, the constant k, and the magnitude of the acceleration due to gravity. [2]

(iv) Show that the equation of motion may be re-expressed as

$$v\frac{dv}{dx} = -g\left(\frac{v^2}{v_T^2} - 1\right),$$

where x is the distance the parachutist has fallen in metres and g is the magnitude of the acceleration due to gravity in m s^{-2}. [3]

(v) In free fall the parachutist gains speed very rapidly, so that when he opens his parachute he is moving with speed much bigger than v_T. Indeed, it is the function of the parachute to reduce his speed to something close to v_T before he reaches the ground. Bearing this in mind, show that the solution of the differential equation in part (iv) appropriate to the situation is

$$v^2 = v_T^2 + e^{-2gx/v_T^2}(v_0^2 - v_T^2),$$

where v is the parachutist's speed when he has fallen distance x with his parachute open, and v_0 is his speed at the instant he opens his parachute. [6]

(vi) Suppose that the effect of a parachute jump, on reaching the ground, should be no worse than the effect of falling from a height of 2 metres under gravity alone. Suppose also that in most jumps the parachutist falls sufficiently far with open parachute that his speed on reaching the ground is effectively his terminal speed. Find what the value of k should be, if the parachutist's mass is 80 kg. [2]

(vii) A jump may be considered safe if the speed on reaching the ground is at most 5% greater than the terminal speed. Suppose that initially the parachutist is at rest, and that the free fall period lasts for 5 seconds. Find the minimum overall height of a safe jump for the parachutist and parachute of part (vi). You may take the value of g to be 10. [5]