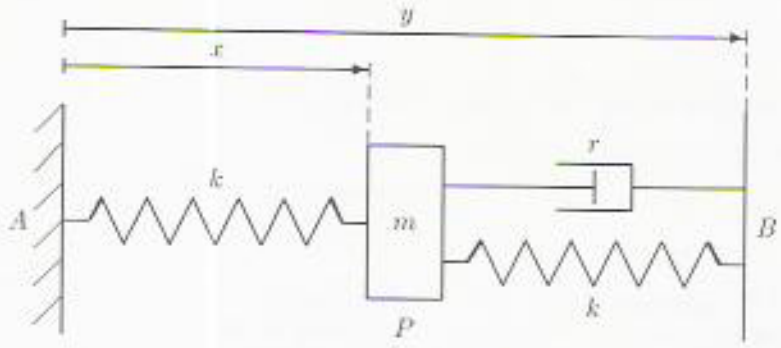
Question 4 (*Unit 8*)



A particle P of mass m slides on a horizontal frictionless track under the influence of two identical perfect springs and a perfect dashpot. The springs each have stiffness k and natural length l_0. The dashpot constant is r. The particle is connected to a fixed point A by one of the springs, and to a forcing point B by the second spring and the dashpot, as shown in the figure. The position of the forcing point B relative to the fixed point A is given by $y(t)$, a known function of time.

(i) Obtain the differential equation for the forced vibrations of the particle relative to the fixed point A, stating clearly the configuration of the system you have assumed in your derivation. [*10*]

The values of the constants, in SI units, are

$$m = 3, \quad k = 12, \quad l_0 = 6, \quad r = 2,$$

and

$$y(t) = 10 + \cos 3t.$$

(ii) Show that, with these values, the equation of motion becomes

$$3\ddot{x} + 2\dot{x} + 24x = 120 + 12\cos 3t - 6\sin 3t.$$ [*2*]

(iii) Find the steady-state solution of this equation *using the methods of solution from Unit 6* (i.e. do not simply quote a formula for the solution from *Unit 8*). Determine the amplitude and phase of the forced vibrations of the particle. [*13*]

$-\cancel{18}$ 48
$-5x_+$