$\ln y = \ln\left(\frac{2(x+1)}{x+2}\right)$ ✓ 2/2

$y = \frac{2(x+1)}{x+2}$ ✓ 2/2

If not given definite limits, we usually use

$\int \frac{1}{y}\frac{dy}{dx} = \int \left(\frac{1}{x+1} - \frac{1}{x+2}\right) dx$

$\therefore \ln y = \ln(x+1) - \ln(x+2) + C$

~~when $x=0$, $y=1$~~

$\ln 1 = \ln 1 - \ln 2 + C$

$\therefore C = \ln 2$

etc. I realise that is the same but this method is more commonly used

b) $x\frac{dy}{dx} = 3y + 3x^6$

~~$x\frac{dy}{dx} - 3y = 3x^6$~~

$\frac{dy}{dx} - \frac{3y}{x} = 3x^5$

Integrating factor is $\exp\left(\int \frac{-3}{x}dx\right) = \exp(-3\ln x) = x^{-3}$ ✓

Multiplying through by integrating factor 3/3

$x^{-3}\frac{dy}{dx} - \frac{3y}{x^4} = 3x^2$

$\frac{d}{dx}(yx^{-3}) = 3x^2$ 1/1

Integrating both sides between x, y and initial conditions

$\int_0^y \frac{d}{dx}(yx^{-3}) = \int_1^x 3x^2 dx$ 2/2

$\left[yx^{-3}\right]_0^y = \left[x^3\right]_1^x$ ✓

$yx^{-3} = x^3 - 1$ ✓ 2/2

OK but same comment as above.

$y = x^6 - x^3$ ✓

c)i) $\frac{dy}{dx} = \sqrt{x} - \sqrt{y}$

writing $\frac{dy}{dx}$ as $\frac{y(x+h) - y(x)}{h} = \frac{y_{r+1} - y_r}{h}$ 2/2

$\therefore \frac{y_{r+1} - y_r}{h} = \sqrt{x_r} - \sqrt{y_r}$

rearranging $y_{r+1} = y_r + h(\sqrt{x_r} - \sqrt{y_r})$.

I prefer: $Y_{r+1} = Y_r + 0.1(\sqrt{x_r} - \sqrt{Y_r})$

Y_r is the approx. value of y_r.

$y(1) = 0.5$ (ie $y_1 = 0.5$)

Also $x_{r+1} = x_r + 0.1$