TMA MST204 01 Part 2 **Cut-off date** 27 February 1997

Questions 2, 3 and 4 below, on *Units 1, 2* and *3*, form the second part of Tutor-marked Assignment MST204 01.

Each question is marked out of 25. Your overall grade will be based on the sum of your marks on these three questions and the question in Part 1.

Please send your answers to Questions 2 to 4 to your tutor. Your tutor should have kept the PT3 for this assignment so that there is no need to send another. (If your tutor has returned your original PT3 by mistake with your answer to Question 1, send it back with your answers to Questions 2 to 4.) You will eventually receive your copy of this PT3, completed by your tutor, along with your answers to these questions.

Question 2 (*Unit 1*)

(i) Find the general solution of the recurrence relation

$$u_{r+1} = -4u_r + 12u_{r-1}.$$

Determine the particular solution of this recurrence relation which satisfies the initial conditions

$$u_0 = \tfrac{1}{6} \quad \text{and} \quad u_1 = -1.$$

Calculate the exact value of u_{10} using your solution. [*10*]

(ii) Use your calculator to compute the values of $u_2, u_3, \ldots, u_{10}$ for the recurrence relation in part (i), using 0.1667 as an approximation to the value of u_0. [*7*]

(iii) By examining the absolute and relative changes in the values of u_{10} resulting from the change from the exact value of u_0 given in part (i) to the approximation for u_0 used in part (ii), decide whether or not the problem of calculating u_{10} using the recurrence relation above with the initial conditions $u_0 = \frac{1}{6}$ and $u_1 = -1$ is

- absolutely ill-conditioned,
- relatively ill-conditioned,

with respect to small changes in the value of u_0. [*8*]

Question 3 (*Unit 2*)

(a) Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{(x+1)(x+2)} \qquad (x > -1)$$

which satisfies the condition $y = 1$ when $x = 0$, expressing your answer in the form $y = f(x)$. [*9*]

(b) Find the solution of the differential equation

$$x\frac{dy}{dx} = 3y + 3x^6 \qquad (x > 0)$$

which satisfies the condition $y = 0$ when $x = 1$, expressing your answer in the form $y = f(x)$. [*8*]

(c) This part of the question is concerned with the use of Euler's method to find an approximate solution of the differential equation

$$\frac{dy}{dx} = \sqrt{x} - \sqrt{y}$$

with initial condition $y(1) = 0.5$.

Question 3 Part (c) continued overleaf.