(iii)

Is the relation absolutely ill conditioned?

$u_{10}' - u_{10} = 10078199.91 - 10077696 = 5039104$

$503.9104 \gg 3.33\times10^{-5} (= u_0' - u_0)$ hence the recurrence relation is absolutely ill conditioned. 4/4

Is the relation relatively ill conditioned?

Compare $\frac{u_{10}' - u_{10}}{u_0}$ and $\frac{u_1' - u_1}{u_1}$

$$\frac{u_{10}' - u_{10}}{u_{10}} = \frac{10078199.91 - 10077696}{10077696} = 5\times10^{-5}$$

$$\frac{u_0' - u_0}{u_0} = \frac{0.1667 - 1/6}{1/6} = 2\times10^{-4}$$

The relative error has fallen from u_0 to u_{10}, hence the recurrence relation is not relatively ill conditioned wrt small changes in u_0.

4/4

OK.

3) $\frac{dy}{dx} = \frac{y}{(x+1)(x+2)}$

rearranging

$$\frac{1}{y}dy = \frac{1}{(x+1)(x+2)}dx$$

Manipulate RHS into linear fractions

$$\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} \Rightarrow 1 = A(x+2) + B(x+1)$$

$x = -1 \Rightarrow 1 = A(-1+2) = A \Rightarrow A = 1$

$x = -2 \Rightarrow 1 = B(-2+1) \Rightarrow B = -1.$

Hence $\frac{1}{y}dy = \left(\frac{1}{x+1} \; \frac{-1}{x+2}\right)dx$ 3/3

Integrating (between y, x and initial conditions).

$$\int_1^y \frac{1}{y}dy = \int_0^x \left(\frac{1}{x+1} - \frac{1}{x+2}\right)dx$$

$$\left[\ln y\right]_1^y = \left[\ln(x+1) - \ln(x+2)\right]_0^x$$ 2/2

$$\ln y - \ln 1 = \ln(x+1) - \ln(x+2) - \ln 1 + \ln 2$$