IST 204 TMA I PART II

1) $u_{r+1} = -4u_r + 12u_{r-1}$

Put $u_n = Ax^n$, then equation becomes

$Ax^{n+1} = -4Ax^n + 12Ax^{n-1}$

Rearranging and dividing through by Ax^{n-1}

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

ie $x = -6$ or $x = 2$

OK. – but just say "Auxiliary Equ" instead

u_n is the sum of solutions

So $u_n = A(-6)^n + B(2)^n$ ✓ 5/5

if $u_0 = 1/6$ and $u_1 = -1$

$u_0 = 1/6 = A(-6)^0 + B(2)^0$

$1/6 = A + B \Rightarrow 1 = 6A + 6B$ ①

$u_1 = A(-6)^1 + B(2)^1 = -1$

$-1 = -6A + 2B$ ✓ ②

① + ②

$0 = 8B \Rightarrow B = 0$ ✓

Hence from either ① or ②, $A = 1/6$ ✓

So $u_n = 1/6(-6)^n$ or $= -(\frac{1}{6})(-6)^n = -(-6)^{n-1}$ ✓ 4/4

$u_{10} = 1/6(-6)^{10} = 1/6 \times 60466176 = 10077696$ ✓ 1/1

ii) Using $u_0 = 0.1667$ and $u_1 = -1$

$u_2 = -4u_1 + 12u_0 = -4 \times -1 + 12 \times 0.1667 = 6.0004$

$u_3 = -4u_2 + 12u_1 = -4 \times 6.0004 + 12 \times -1 = -36.0016$

$u_4 = -4u_3 + 12u_2 = -4 \times -36.0016 + 12 \times 6.0004 = 216.0112$

$u_5 = -4u_4 + 12u_3 = -4 \times 216.0112 + 12 \times -36.0016 = -1296.064$

$u_6 = -4u_5 + 12u_4 = -4 \times -1296.064 + 12 \times 216.0112 = 7776.3904$

$u_7 = -4u_6 + 12u_5 = -4 \times 7776.3904 + 12 \times -1296.064 = -46658.3296$

$u_8 = -4u_7 + 12u_6 = -4 \times -46658.3296 + 12 \times 7776.3904 = 279950.00$

$u_9 = -4u_8 + 12u_7 = -4 \times 279950.0032 + 12 \times -46658.3296 = -1679699.9$

$u_{10} = -4u_9 + 12u_8 = -4 \times 1679699.968 + 12 \times 279950.0032 = 10078199.$

No need to show me all of this when using a calculator 7/7