$$\frac{dP}{dt} = P\left(0.2 - \frac{P}{2E4}\right) = P\left(0.2 - \frac{0.2P}{4000}\right)$$

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{4000}\right)$$ fair enough 4/4

DIVISION SIGNS

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{4000}\right) = \frac{P}{5}\left(\frac{4000-P}{4000}\right) = \frac{P(4000-P)}{20,000}$$

$$\frac{20000}{P(4000-P)}\,dP = dt$$

Manipulate LHS into linear fractions

$$\frac{20000}{P(4000-P)} = \frac{A}{P} + \frac{B}{4000-P}$$

$$20000 = A(4000-P) + BP$$

Put $P = 4000$

$$20000 = 0 + B \times 4000 \Rightarrow B = 5$$

Put $P = 6000$

$$20000 = -2000A + 6000 \times 5$$

$$-10000 = -2000A \Rightarrow A = 5$$

$$\frac{20000}{P(4000-P)}\,dP = \left(\frac{5}{P} + \frac{5}{4000-P}\right)dP = dt$$

If you find the Logistics model – then you can just quote the solution on page 19 UNIT 3 or in the Handbook

New stable population given by soln to $\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{4000}\right)$ for $\frac{dP}{dt} = 0$ ie $P = 0$ or $P = 4000$ 1/1

We're not interested in $P = 0$, only $P = 4000$.

Integrate $\left(\frac{5}{P} + \frac{5}{4000-P}\right)dP = dt$ between $P = 4200$ and $P = 6000$, and $t = t$, $t = 0$

3/3

$$\int_{6000}^{4200}\left(\frac{5}{P} + \frac{5}{4000-P}\right)dP = \int^{t} dt$$

$$\Big[5\ln P - 5\ln(4000-P)\Big]_{6000}^{4200} = t$$

$$t = \left[5\ln\left(\frac{P}{4000-P}\right)\right]_{6000}^{4200}$$

$$= 5\ln\left(\frac{4200}{4000-4200}\right) - 5\ln\left(\frac{6000}{4000-6000}\right)$$