(i) Write down the recurrence relation for Euler's numerical method applied to the above problem. Use this recurrence relation with a step length $h = 0.1$ to calculate approximations to $y(1.1)$ and $y(1.2)$. Give your answers correct to six decimal places. [4]

(ii) Euler's method was applied to the above problem on a computer, with various step lengths h, to calculate approximate values of $y(1.2)$. The results are shown in the table below.

Step length h	Approximation to $y(1.2)$
0.001	0.563 931 5
0.002	0.563 907 4
0.004	0.563 859 2
0.005	0.563 835 0
0.008	0.563 762 4
0.01	0.563 713 9

Plot a graph of these approximations to $y(1.2)$ against step length h. What does your graph indicate to be the relationship between the Euler approximations to $y(1.2)$ and the step length h for small h? Use your graph to estimate the true value of $y(1.2)$ correct to six decimal places. [4]

$D = 0.2P + m(P-2000)$

$400 = 750 - 600 + m(1000)$

Question 4 (*Unit 3*)

A colony of geese has had a stable population for some years. Prior to this period of stability, the population was growing, initially from a low level. When the population was 2000, the proportionate birth rate was 40% per year and the proportionate death rate was 20% per year. When the population was 3000, the proportionate birth rate remained at 40% per year, whereas the proportionate death rate had risen to 25% per year.

(i) A continuous (i.e. differential equation) model of the growth of the colony is constructed using the following assumptions.

- The proportionate birth rate is constant.
- The proportionate death rate is an increasing linear function of population size.
- There is no migration.
- There is no exploitation of the colony.

Using the above assumptions and data, derive expressions for the proportionate birth rate and the proportionate death rate in terms of the population size P of the colony. [5]

Hence find the differential equation satisfied by the population of the colony during its growth. Further, determine the present stable size of the colony. [6]

(ii) Due to environmental changes, it is predicted that in future emigration from the colony will occur at the rate of 10% of the population per year. It is not expected that the proportionate birth and death rates will be affected. Show that the differential equation which may be used to model the future behaviour of the population is

$$\frac{dP}{dt} = \tfrac{1}{5}P\left(1 - \frac{P}{4000}\right).$$ [4]

Use this equation to predict how long it will be before the population falls from its present stable size to within 5% of its new equilibrium level. [7]

(iii) Sketch a graph showing how the population has grown to its present level and how it is predicted that the population will change in the future. [3]

You may use, without proof, any formula given in the *Handbook*.