

Options for Questions 13 to 15

- A $\frac{\partial U}{\partial x} + 3x \frac{\partial^2 U}{\partial t^2} = e^t$
 B $x \frac{\partial^2 U}{\partial x^2} + 7y \frac{\partial^2 U}{\partial x \partial y} + 6 \frac{\partial^2 U}{\partial y^2} = 5$
 C $\frac{\partial^2 U}{\partial x^2} + 3 \frac{\partial^2 U}{\partial y^2} = 0$
 D $\frac{\partial^2 U}{\partial x^2} + 3 \frac{\partial^2 U}{\partial x \partial y} + x \frac{\partial U}{\partial x} = x$
 E $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + 3 \frac{\partial U}{\partial x} + 2U = \sin(x+y)$
 F $\frac{\partial^2 U}{\partial x^2} + \frac{\partial U}{\partial x} \frac{\partial U}{\partial t} + 3tU = \sin t$

[Question 15 has TWO correct options.]

Questions 16 to 24

Consider the problem of solving the partial differential equation

$$\frac{\partial^2 U}{\partial x^2} - \frac{\partial U}{\partial t} - U = 0 \quad (0 < x < \pi, t > 0)$$

for the function $U(x, t)$, subject to the conditions

$$U(0, t) = 0 \quad (t \geq 0),$$

$$U(\pi, t) = 0 \quad (t \geq 0),$$

$$U(x, 0) = \begin{cases} \frac{2x}{\pi} & (0 \leq x \leq \frac{1}{2}\pi), \\ 2 - \frac{2x}{\pi} & (\frac{1}{2}\pi < x \leq \pi), \end{cases}$$

by the method of separation of variables.

- 16 Select the option which gives a pair of ordinary differential equations for the two functions $X(x)$ and $T(t)$ which will ensure that the function $U(x, t) = X(x)T(t)$ satisfies the given partial differential equation. In the options, μ is a constant.

Options

- A $X'' = \mu T, T' = (\mu + 1)X$ B $X'' = \mu X, T' = (\mu + 1)T$
 C $X'' = \mu T, T' = (\mu - 1)X$ ☒ D $X'' = \mu X, T' = (\mu - 1)T$ D
 E $X'' = \mu T, T' = -(\mu + 1)X$ ☒ F $X'' = \mu X, T' = -(\mu + 1)T$
 G $X'' = \mu T, T' = -(\mu - 1)X$ H $X'' = \mu X, T' = -(\mu - 1)T$

- 17 Select the option which gives the values of μ which lead to non-trivial solutions $U(x, t)$ of the given partial differential equation compatible with the given boundary conditions when $U(x, t) = X(x)T(t)$ with $X(x)$ and $T(t)$ solutions of the equations in the correct option for Question 16.

Options

- ☒ A $0, 1, 2, 3, \dots, n, \dots$ ☒ B $1, 2, 3, 4, \dots, n, \dots$
 C $0, -1, -2, -3, \dots, -n, \dots$ ☒ D $-1, -2, -3, -4, \dots, -n, \dots$
 E $0, 1, 4, 9, \dots, n^2, \dots$ ☒ F $1, 4, 9, 16, \dots, n^2, \dots$
 G $0, -1, -4, -9, \dots, -n^2, \dots$ ☒ H $-1, -4, -9, -16, \dots, -n^2, \dots$

$$\frac{X''}{X} = \mu, -1$$