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d) From a) $\theta - \frac{\Omega t}{2} = \phi, J=1$

$$\therefore \theta = \phi + \frac{\Omega t}{2} = 1 - \gamma$$

$$q = \sqrt{2I} \sin \theta = \sqrt{2(1-\gamma)} \sin\left(\frac{\Omega t}{2} + \phi\right)$$

$$p = \sqrt{2I} \cos \theta = \sqrt{2(1-\gamma)} \cos\left(\frac{\Omega t}{2} + \phi\right)$$

where $\gamma = \frac{4}{\epsilon} \left(1 - \frac{\Omega}{2}\right)$

In c) we expanded the Hamiltonian around the stable fixed point

$$K = \frac{\epsilon(1-\gamma)}{2} (\phi - \phi_f)^2 + \frac{\epsilon}{8} (J - J_f)^2$$

where $4\phi_f = -\pi \Rightarrow \phi_f = -\frac{\pi}{4}$

Put $J = 1 - \gamma$
 $\phi - \phi_f = X \Rightarrow \dot{\phi} = \dot{X}$

$$K = \frac{\epsilon(1-\gamma)}{2} X^2 + \frac{\epsilon}{8} Y^2 \Rightarrow \dot{J} = \dot{Y}$$

$$\dot{X} = \frac{\partial K}{\partial Y} = \frac{\epsilon}{4} Y \quad \dot{Y} = -\frac{\partial K}{\partial X} = -\epsilon(1-\gamma)X$$

$$\ddot{X} = \frac{\epsilon}{4} \dot{Y} \Rightarrow \ddot{X} = -\frac{\epsilon^2}{4} (1-\gamma) X$$

$$X + \frac{\epsilon^2}{4} (1-\gamma) X = 0$$

$$X = A \sin\left(\frac{\epsilon t \sqrt{1-\gamma}}{2} + \delta\right)$$

$$\dot{Y} = -\epsilon(1-\gamma)X = -\epsilon(1-\gamma)A \sin\left(\frac{\epsilon t \sqrt{1-\gamma}}{2} + \delta\right)$$

$$Y = 2\sqrt{1-\gamma} A \cos\left(\frac{\epsilon t \sqrt{1-\gamma}}{2} + \delta\right)$$

$$\phi = \phi_f + X = -\frac{\pi}{4} + A \sin\left(\frac{\epsilon t \sqrt{1-\gamma}}{2} + \delta\right)$$

$$J = J_f + Y = (1-\gamma) + 2\sqrt{1-\gamma} A \cos\left(\frac{\epsilon t \sqrt{1-\gamma}}{2} + \delta\right)$$

That

You should state that the stable fixed point of K is equivalent to a stable periodic orbit in the original representation

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