

$$n^2 + 3n + 2 = n^2 + 5n + 2$$

$$0 = 2n \Rightarrow n = 0$$

But $n \geq 1$ $\therefore n = 0$ not possible
 \therefore No iterate can begin with either 0.011 or 0.101

Suppose we have a periodic orbit, so that $b_p = b_{p+k}$, $p = 1, 2, 3, \dots$. The ones occur at $f(n), f(n+1), \dots$. The b_p are not all zero. (If they are all zero then there are no ones beyond a certain point, but this is a contradiction, since $f(m) > f(n)$ for $m > n$, $f'(n) = n + \frac{3}{2} > 0$)

\therefore since the set of natural numbers is unbounded above $f(m)$ is unbounded above and there are an infinite no. of 1s.

The ones are separated by at most $k-1$ zeros. $f(n+r) - f(n)$

$$= \frac{(n+r)^2 + 3(n+r) - 2}{2} - \frac{n^2 + 3n - 2}{2}$$

$$= \frac{n^2 + 2nr + r^2 + 3n + 3r - 2 - n^2 - 3n + 2}{2}$$

$$= \frac{2nr + r^2 + 3r}{2} \text{ is least when } r = 1$$

Put $r = 1$, $f(n+1) - f(n) = \frac{2n + 4}{2} = n + 2$

Since $n \in \mathbb{N}$, n unbounded above we can find n such that $n + 2 > k - 1$ and there are no ones in the $[b_p, b_{p+k}]$ but there are an infinite number of ones \therefore the orbit of E_0 is not periodic.

If the orbit of E_0 were dense the iterates of E_0 would pass arbitrarily close to every point in the interval $[0, 1]$, but no iterate begins with 0.011 (or 0.11) \therefore no iterates pass arbitrarily close to 1 \therefore the orbit of E_0 is not dense. The same argument applies

(12/3)

Obviously not periodic since no repetition.

(2/5)