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$$(F^2(x))' = \left(2\cdot\left(\frac{-(\lambda+1)-\sqrt{(\lambda+1)(\lambda-3)}}{2}\right)+\lambda\right)\left(2\left(\frac{-(\lambda+1)+\sqrt{(\lambda+1)(\lambda-3)}}{2}\right)+\lambda\right)$$

$$= \left(-1-\sqrt{(\lambda+1)(\lambda-3)}\right)\left(-1+\sqrt{(\lambda+1)(\lambda-3)}\right)$$

$$= 1-(\lambda+1)(\lambda-3) = 1-(\lambda^2-2\lambda-3)$$

$$= -\lambda^2+2\lambda+4$$

$$-1 < -\lambda^2+2\lambda+4 < 1$$

Take the left hand inequality 1st.

$$-1 < -\lambda^2+2\lambda+4$$

$$\lambda^2-2\lambda-5<0$$

$$\lambda^2-2\lambda-5=0$$

$$\lambda = \frac{2\pm\sqrt{4-4\cdot1\cdot-5}}{2}$$

$$= \frac{2\pm\sqrt{24}}{2} = 1\pm\sqrt{6}$$

$$(\lambda-(1+\sqrt{6}))(\lambda-(1-\sqrt{6}))<0$$

$$\lambda < 1+\sqrt{6}, \quad \lambda > 1-\sqrt{6}$$

ie $1-\sqrt{6}<\lambda<1+\sqrt{6}$

The right hand inequality gives

$$-\lambda^2+2\lambda+4<1$$

$$0<\lambda^2-2\lambda-3$$

$$0<(\lambda-3)(\lambda+1) \Rightarrow \lambda>3 \text{ or } \lambda<-1$$

together with the range of values of λ given above this means

$$3<\lambda<1+\sqrt{6}$$

or $1-\sqrt{6}<\lambda<-1$

$\lambda>0 \Rightarrow 3<\lambda<1+\sqrt{6} \Rightarrow \lambda=1+\sqrt{6}$ is the value of λ at which period doubling from period 2 to 4 occurs.

$\frac{6}{6}$

Alternatively

Sub $x_n = ay_n+b$

$$ay_{n+1}+b = (ay_n+b)^2+a\lambda y_n+b\lambda$$

$$= a^2y_n^2+2aby_n+b^2+a\lambda y_n+b\lambda$$

$$ay_{n+1} = a^2y_n^2+ay_n(2b+\lambda)+b(b+\lambda-1)$$

$$y_{n+1} = a\cdot y_n^2+y_n(2b+\lambda)+\frac{b}{a}(b+\lambda-1)$$

To convert to form $y_{n+1}=y_n^2+c$

Put $a=1$ $b=-\frac{\lambda}{2}$