③

The phase curves are curves of constant K.

$$K_c = J\left(1 + \frac{\varepsilon}{4}\sin 2\phi - \frac{\Omega}{2}\right)$$

$$J = K_c\left(1 + \frac{\varepsilon}{4}\sin 2\phi - \frac{\Omega}{2}\right)^{-1}$$

If $-\frac{\varepsilon}{4} < 1 - \frac{\Omega}{2} < \frac{\varepsilon}{4}$

the bracketed term has real roots at which J becomes infinite $\therefore$ this happens when $\Omega \simeq 2$ $\therefore$ there is a resonance at $\Omega = 2$

b) From ① deduced in part ⓐ

$$K(\phi, J, t) = J + \frac{\varepsilon J^2}{8} + \frac{\varepsilon J}{4}\sin 2\phi - \frac{\Omega J}{2}$$
$$+ \frac{\varepsilon J}{8}\left(4\sin\Omega t - J\cos(4\phi + 2\Omega t) - 2\sin 2(\phi + \Omega t)\right)$$

The mean motion Hamiltonian is found by integrating over a period of duration $\frac{2\pi}{\Omega}$; since ϕ varies little over this time we can keep this constant.

This is one way of looking at it — See other as opposite

$$\bar{K} = \frac{\Omega}{2\pi}\int_{t-\pi/\Omega}^{t+\pi/\Omega} \left(J + \frac{\varepsilon J^2}{8} + \frac{\varepsilon J}{4}\sin 2\phi - \frac{\Omega J}{2}\right.$$
$$\left. + \frac{\varepsilon J}{8}\left(4\sin\Omega t - J\cos(4\phi + 2\Omega t) - 2\sin 2(\phi + \Omega t)\right)\right)dt$$

$$= \frac{\Omega}{2\pi}\left[\left(J + \frac{\varepsilon J^2}{8} + \frac{\varepsilon J}{4}\sin 2\phi - \frac{\Omega J}{2}\right)t\right]_{t-\pi/\Omega}^{t+\pi/\Omega}$$

$$+ \frac{\Omega}{2\pi}\left[\frac{\varepsilon J}{8}\left(\frac{4\cos\Omega t}{\Omega} - \frac{J}{2\Omega}\sin(4\phi + 2\Omega t) + \frac{1}{\Omega}\cos 2(\phi + \Omega t)\right)\right]_{t-\frac{\pi}{\Omega}}^{t+\frac{\pi}{\Omega}}$$

No need for all of this

$$= \frac{\Omega}{2\pi}\left(J + \frac{\varepsilon J^2}{8} + \frac{\varepsilon J}{4}\sin 2\phi - \frac{\Omega J}{2}\right)\cdot\frac{2\pi}{\Omega}$$
$$+ \frac{\Omega\varepsilon J}{16\pi}\left(-\frac{4}{\Omega}\left(\cos\Omega(t + \pi/\Omega) - \cos\Omega(t - \pi/\Omega)\right)\right.$$
$$- \frac{J}{2\Omega}\left(\sin(4\phi + 2\Omega(t + \pi/\Omega)) - \sin(4\phi + 2\Omega(t - \pi/\Omega))\right)$$
$$\left. + \frac{1}{\Omega}\left(\cos(2\phi + 2\Omega(t + \pi/\Omega)) - \cos(2\phi + 2\Omega(t - \pi/\Omega))\right)\right)$$

$$= J + \frac{\varepsilon J^2}{8} + \frac{\varepsilon J}{4}\sin 2\phi - \frac{\Omega J}{2}$$

This could be written down immediately