2) $x_{n+1} = x_n(x_n+\lambda) = F(x_n)$

$F^2(x_n) = x_{n+1}(x_{n+1}+\lambda)$

$\quad = x_n(x_n+\lambda)(x_n(x_n+\lambda)+\lambda)$

$x_n = F^2(x_n)$ for a fixed period two point

Put $x_n = x$

$x(x+\lambda)(x^2+x\lambda+\lambda) = x$

$x((x+\lambda)(x^2+x\lambda+\lambda)-1) = 0$

$x(x^3+x^2\lambda+x\lambda+x^2\lambda+x\lambda^2+\lambda^2-1) = 0$

$x(x^3+2x^2\lambda+x(\lambda^2+\lambda)+\lambda^2-1) = 0$

Find the fixed points of $F(x)$, given by $x = F(x)$

$x(x+\lambda) = x$

$x^2+x\lambda-x = 0$

$x(x+\lambda-1) = 0 \Rightarrow x = 0,\ x = 1-\lambda$

These values of x are also roots to $F^2(x) = x$ ie $x+\lambda-1$ is a factor

$$
\begin{array}{r l}
 & x^2+x(\lambda+1)+\lambda+1 \\
x+\lambda-1 & \overline{)\,x^3+2x^2\lambda+x(\lambda^2+\lambda)+\lambda^2-1} \\
 & x^3+x^2(\lambda-1) \\
 & x^2(\lambda+1) \\
 & x^2(\lambda+1)+x(\lambda^2-1) \\
 & x(\lambda+1) \\
 & x(\lambda+1)+\lambda^2-1
\end{array}
$$

The fixed points of $F^2(x)$ which are not fixed points of F are given by the eqtn

$x^2+x(\lambda+1)+\lambda+1 = 0$

$x = \dfrac{-(\lambda+1) \pm \sqrt{(\lambda+1)^2-4(\lambda+1)}}{2}$

$\quad = \dfrac{-(\lambda+1) \pm \sqrt{(\lambda+1)(\lambda+1-4)}}{2}$

$\quad = \dfrac{-(\lambda+1) \pm \sqrt{(\lambda+1)(\lambda-3)}}{2}$

These fixed points are stable if $-1 < (F^2(x))' < 1$

but $(F^2(x))' = F'(x_1)F'(F(x_1))$

$\quad = F'(x_1)F'(x_2)$

$\quad = (2x_1+\lambda)(2x_2+\lambda)$

where $x_1 = \dfrac{-(\lambda+1) + \sqrt{(\lambda+1)(\lambda-3)}}{2}$

$x_2 = \dfrac{-(\lambda+1) - \sqrt{(\lambda+1)(\lambda-3)}}{2}$