What is r?

21

The separation between islands is
$\Delta I_n = |I_{n+1} - I_n| \approx \frac{\partial I_n}{\partial n}$ Notation shaky?

The condition for islands not to intersect is that
$\Delta I_n \approx \frac{\partial I_n}{\partial n} > \frac{1}{2}(s^{n+1} + s^n) \approx \frac{1}{2}(s^n + s^n) = (s^n)$

$\frac{\partial I_n}{\partial n} = \frac{\partial}{\partial n}\left(\frac{\pi^2 a^2}{3m}\frac{n^3}{s^3}\right) = \frac{\pi^2 a^2 n^2}{m s^3}$

thus $\frac{\pi^2 a^2 n^2}{m s^3} > \frac{4a\pi n^2}{m s^3}\sqrt{aF}$

$\sqrt{aF} < \frac{\pi^2 a^2 n^2}{m s^3} \cdot \frac{m s^3}{4a\pi n^2} = \frac{\pi a}{4}$

$aF < \frac{\pi^2 a^2}{16} \Rightarrow F < \frac{\pi^2 a}{16}$

Hence if $F < \frac{\pi^2 a}{16}$, the resonance islands do not overlap, and the approximation is valid; if $F \geq \frac{\pi^2 a}{16}$ the resonance islands overlap and the approximation is not valid. ✓

3/4

$$\Delta I_n = |I_{n+1} - I_n| \approx \frac{dI_n}{dn}.$$

Total 23/25