TMA MS323 04

1) $H(q,p,t) = \frac{1}{2}(p^2+q^2) + \frac{1}{4}\varepsilon q^2 p^2 + \frac{1}{2}\varepsilon q^2 \sin\Omega t$

$p = \sqrt{2I m\omega}\cos\Theta \quad q = \sqrt{\frac{2I}{m\omega}}\sin\Theta$

$m = \omega = 1 \therefore p = \sqrt{2I}\cos\Theta, \; q = \sqrt{2I}\sin\Theta$ ✓

$H(\Theta, I, t) = \frac{1}{2}((\sqrt{2I}\cos\Theta)^2 + (\sqrt{2I}\sin\Theta)^2)$

$+ \frac{1}{4}\varepsilon(\sqrt{2I}\sin\Theta)^2(\sqrt{2I}\cos\Theta)^2$

$+ \frac{1}{2}\varepsilon(\sqrt{2I}\sin\Theta)^2\sin\Omega t$

$= \frac{1}{2}\cdot 2I(\sin^2\Theta + \cos^2\Theta) + \frac{1}{4}\varepsilon(2I\cdot 2I\cdot\sin^2\Theta\cdot\cos^2\Theta)$

$+ \frac{1}{2}\cdot\varepsilon\cdot 2I\sin^2\Theta\sin\Omega t$

$= I + \frac{\varepsilon}{4}(2I\sin\Theta\cos\Theta)^2 + \varepsilon I\sin^2\Theta\sin\Omega t$

$= I + \frac{\varepsilon I^2}{4}\sin^2 2\Theta + \varepsilon I\sin^2\Theta\sin\Omega t$ ✓

$\cos 2\Theta = 1 - 2\sin^2\Theta$

$\therefore -2\sin^2\Theta = 1 - \cos 2\Theta \Rightarrow \sin^2\Theta = \frac{1}{2}(1-\cos 2\Theta)$

Similarly $\sin^2 2\Theta = \frac{1}{2}(1-\cos 4\Theta)$

$\therefore H(\Theta, I, t) = I + \frac{\varepsilon I^2}{4}\cdot\frac{1}{2}(1-\cos 4\Theta)$

$+ \varepsilon I\cdot\frac{1}{2}(1-\cos 2\Theta)\sin\Omega t$

$= I + \frac{\varepsilon I^2}{8} - \frac{\varepsilon I^2}{8}\cos 4\Theta + \frac{\varepsilon I}{2}\sin\Omega t$

$- \frac{\varepsilon I}{2}\cos 2\Theta\sin\Omega t$

$\cos 2\Theta\sin\Omega t = \frac{1}{2}(\sin(2\Theta+\Omega t) - \sin(2\Theta-\Omega t))$ ✓

$\therefore H(\Theta, I t) = I + \frac{\varepsilon I^2}{8} - \frac{\varepsilon I^2}{8}\cos 4\Theta + \varepsilon I\sin\Omega t$

$- \frac{\varepsilon I}{2}\cdot\frac{1}{2}(\sin(2\Theta+\Omega t) - \sin(2\Theta-\Omega t))$

$= I + \frac{\varepsilon I^2}{8} - \frac{\varepsilon I^2}{8}\cos 4\Theta + \frac{\varepsilon I}{2}\sin\Omega t$

$- \frac{\varepsilon I}{4}\sin(2\Theta+\Omega t) + \frac{\varepsilon I}{4}\sin(2\Theta-\Omega t)$