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c) The Linearisation of the map around a point (a,b) is the Jacobian matrix

$$L(a,b) = \begin{vmatrix} \frac{\partial X}{\partial x} & \frac{\partial X}{\partial y} \\ \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial y} \end{vmatrix}_{(a,b)}$$

Where $X = y$
$Y = -x + cy(y-1)$

$$L(a,b) = \begin{bmatrix} 0 & 1 \\ -1 & 2cy - c \end{bmatrix}_{(a,b)} \Rightarrow \det(L(a,b)) = 0\cdot(2ca - c) - -1 = 1$$

∴ the map is area preserving ✓

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ii) The fixed points are the solns to
$x = y$
$y = -x + cy(y-1)$

ie $y = -y + cy^2 - cy$
$0 = cy^2 - 2y - cy = y(cy - 2 - c)$

The solns are $y = 0$ $y = \frac{2+c}{c}$

draw in the fraction lines!

Then $x = y$ so the fixed points are $(0,0)$, $\left(\frac{2+c}{c}, \frac{2+c}{c}\right)$

A fixed point is stable if $|\mathrm{tr}(L(a,b))| < 2$

See over.

$\mathrm{tr}\, L(0,0) = 0 + 2c\cdot 0 - c = -c$

ie $|-c| < 1$
$-1 < -c < 1$
$1 > c > -1$

ie fixed point at $(0,0)$ stable if $-1 < c < 1$. X

X

$\mathrm{tr}\, L\left(\frac{2+c}{c}, \frac{2+c}{c}\right) = 0 + 2c\left(\frac{2+c}{c}\right) - c$
$= 4 + 2c - c = 4 + c$

$|4+c| < 1$
$-1 < 4 + c < 1$
$-5 < c < -3$

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ie the fixed point at $\left(\frac{2+c}{c}, \frac{2+c}{c}\right)$ is stable if $-5 < c < -3$ X

Total 18/25