$$Q_n = \frac{2}{\pi}\int_0^{\pi} \frac{I^{2/3}A}{a}\left(\frac{2\theta}{\pi} - \frac{\theta^2}{\pi^2}\right)\cos n\theta \, d\theta$$

$$\frac{2I^{2/3}A}{\pi a}\left[\int_0^{\pi} \frac{2\theta \cos n\theta \, d\theta}{\pi} - \int_0^{\pi} \frac{\theta^2}{\pi^2} \cos n\theta \, d\theta\right]$$

$$\frac{2I^{2/3}A}{\pi a}\left[\frac{2}{\pi}\int_0^{\pi} \theta \cos n\theta \, d\theta - \frac{1}{\pi^2}\int_0^{\pi} \theta^2 \cos n\theta \, d\theta\right]$$

Using the integrals given in the question this becomes

$$\frac{2I^{2/3}A}{\pi a}\left[\frac{2(\cos n\pi - 1)}{\pi n^2} - \frac{2\pi}{\pi^2 n^2}\cos n\pi\right]$$

$$\frac{2I^{2/3}A}{\pi a}\left(\frac{2\cos n\pi}{\pi n^2} - \frac{2}{\pi n^2} - \frac{2}{\pi n^2}\cos n\pi\right)$$

$$= \frac{-4I^{2/3}A}{\pi^2 n^2 a} = \frac{-4A}{a\pi^2 n^2} I^{2/3} = Q_n \quad ✓$$

$\frac{6}{6}$

c) The Hamiltonian has the form $H(q,p,t) = H_0(q,p) + Fq\cos\Omega t$, where from a) $H_0(q,p) = E(I) = AI^{2/3}$

The nth resonance action is given by $n\omega(I_n) = \Omega$ where $\omega(I_n) = \frac{\partial H_0}{\partial I}$

$$\omega(I_n) = \frac{\partial H_0}{\partial I} = \frac{\partial}{\partial I}(AI^{2/3}) = \frac{2A}{3I^{1/3}}$$

$$\text{ie} \quad \frac{2nA}{3I^{1/3}} = \Omega \Rightarrow I^{1/3} = \frac{2nA}{3\Omega} = \frac{2A}{3}\left(\frac{n}{\Omega}\right)$$

$$I^{1/3} = \frac{2}{3}\left(\frac{3\pi a}{2\sqrt{2m}}\right)^{2/3}\left(\frac{n}{\Omega}\right)$$

$$I_n = \frac{8}{27}\cdot\frac{9\pi^2 a^2}{8m}\left(\frac{n}{\Omega}\right)^3 = \frac{\pi^2 a^2}{3m}\left(\frac{n}{\Omega}\right)^3 \quad ✓$$

Expressed in terms of the angle action variables, this becomes