ii) The number ξ_0 can be expressed as $\frac{1}{2^1}+\frac{1}{2^4}+\frac{1}{2^8}+\frac{1}{2^{13}}$

1st power is 1
2nd power is $4 = 1+(1+2)$
3rd power is $8 = 1+(1+2)+(1+3) = 2+(1+2+3)$
4th power is $13 = 1+(1+2)+(1+3)+(1+4) = 3+(1+2+3+4)$
ie the nth power is $(n-1)+(1+2+3\cdots+n)$

$= n-1+\frac{n(2\cdot 1+(n-1)\cdot 1)}{2}$

$= n-1+\frac{n(2+n-1)}{2} = n-1+\frac{n(n+1)}{2}$

$= \frac{(2n-2+n^2+n)}{2} = \frac{(n^2+3n-2)}{2}$

Then $\xi_0 = \sum_{n=1}^{\infty} \frac{1}{2^{\frac{(n^2+3n-2)}{2}}}$

Put $f(n) = \frac{n^2+3n-2}{2}$

$\xi_0 = \sum_{n=1}^{\infty} \frac{1}{2^{f(n)}}$

Your work is very difficult to read. and your argument difficult to follow here

For an iterate to begin with either (1) 0.011 or (2) 0.101 we must have for some $f(n), f(n+1)$

$\frac{1}{2\cdot 2^{f(n)}} = \frac{1}{2^{f(n+1)}} \Rightarrow f(n)+1 = f(n+1)$ (1)

I don't follow this step. You haven't explained.

or $2^2\cdot 2^{f(n)} = 2^{f(n+1)} \Rightarrow f(n)+2 = f(n+1)$ (2)

Similarly here.

(1) $f(n)+1 = f(n+1)$

$\frac{n^2+3n-2}{2}+1 = \frac{(n+1)^2+3(n+1)-2}{2}$

$\frac{n^2+3n}{2} = \frac{n^2+5n+2}{2}$

$n^2+3n = n^2+5n+2$

$-2 = 2n \Rightarrow n=-1$

But $n \in N^+ \therefore n \geq 1 \therefore n=-1$ is not possible.

(2) $f(n)+2 = f(n+1)$

$\frac{n^2+3n-2}{2}+2 = \frac{(n+1)^2+3(n+1)-2}{2}$

$\frac{n^2+3n+2}{2} = \frac{n^2+5n+2}{2}$