Then $K(\phi, X) = \frac{1}{2}\frac{\partial^2 H_0}{\partial I^2}(I_n)X^2 - \frac{2FAI_n^{2/3}}{a\pi^2 n^2}\cos n\phi$

$\frac{\partial H_0}{\partial I} = \frac{\partial}{\partial I}(AI^{2/3}) = \frac{2}{3}AI^{-1/3}$

Retaining only second order terms

$\frac{\partial^2 H_0}{\partial I^2} = \frac{\partial}{\partial I}\left(\frac{2}{3}AI^{-1/3}\right) = -\frac{2}{9}AI^{-4/3}$

$\therefore \frac{1}{2}\frac{\partial^2 H_0}{\partial I^2}(I_n) = -\frac{AI_n^{-4/3}}{9}$

$\therefore K(\phi, X) = \frac{-A}{9I_n^{4/3}}X^2 - \frac{2FAI_n^{2/3}}{a\pi^2 n^2}\cos n\phi$

Where because the perturbation (it is assumed is small) we can ignore terms $O(X^1)$ in expanding the second term.

But how do you know this?

To find the separatrix width we first have to find the separatrix energy. This is done by setting $X = 0$ and $n\phi = 0$. The separatrix energy is then $E = -\frac{2AFI_n^{2/3}}{a\pi^2 n^2}$. The eqtn of the separatrix is

$-\frac{2AFI_n^{2/3}}{a\pi^2 n^2} = \frac{-A}{9I_n^{4/3}}X^2 - \frac{2AFI_n^{2/3}}{a\pi^2 n^2}\cos n\phi$

$\frac{-AX^2}{9I_n^{4/3}} = -\frac{2AFI_n^{2/3}}{a\pi^2 n^2}(1 - \cos n\phi)$

$X^2 = \frac{18FI_n^2}{a\pi^2 n^2}(1 - \cos n\phi)$

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The largest value of X^2 occurs when $\cos n\phi = -1$ ($n\phi = \pi$)

$X^2 = \frac{36FI_n^2}{a\pi^2 n^2}$

$X = \sqrt{\frac{36FI_n^2}{a\pi^2 n^2}} = \frac{6I_n}{\pi n}\sqrt{\frac{F}{a}}$

Sub $I_n = \frac{\pi^2 a^2}{3m}\left(\frac{n}{\Omega}\right)^3$

$X_{max} = \frac{6}{\pi n} \cdot \frac{\pi^2 a^2}{3m}\left(\frac{n}{\Omega}\right)^3\sqrt{\frac{F}{a}} = \frac{2\pi a^2 n^2}{m\Omega^3}\sqrt{\frac{F}{a}} = \frac{2\pi a n^2\sqrt{aF}}{m\Omega^3}$

The separatrix width is twice this maximum value of X

$S^n = \frac{4\pi a n^2\sqrt{aF}}{m\Omega^3}$