$$0 = \frac{\varepsilon}{4}(\gamma + \sin 2\phi + J) \Rightarrow \gamma + \sin 2\phi + J = 0 \quad (1)$$

$$0 = -\frac{\varepsilon J}{2}\cos 2\phi \Rightarrow \cos 2\phi = 0 \quad (2)$$

$(2) = 0$ when $2\phi = (n + 1/2)\pi$

From (1) $J = -\gamma - \sin 2\phi$
$= -\gamma - \sin(n + 1/2)\pi.$
$= -\gamma - (\sin n\pi \cos \pi/2 + \cos n\pi \sin \pi/2)$
$= -\gamma - (-1)^n$

$n = -1 \quad J = -\gamma - (-1) = 1 - \gamma$
$2\phi = (-1 + 1/2)\pi = -\pi/2 \Rightarrow 4\phi = -\pi$

Are linearisation
~~Linearise~~ the mean motion Hamiltonian around this fixed point

these are 2nd order conditions so you are not linearising

$$\frac{\partial^2 K}{\partial \phi^2} = \frac{\varepsilon J}{2}\frac{\partial}{\partial \phi}(\cos 2\phi) = -\varepsilon J \sin 2\phi$$

$$\frac{\partial^2 K}{\partial J^2} = \frac{\varepsilon}{4}\frac{\partial}{\partial J}(\gamma + \sin 2\phi + J)$$

$$= \frac{\varepsilon}{4} \cdot 1 = \frac{\varepsilon}{4}$$

$$\frac{\partial^2 K}{\partial J \partial \phi} = -\frac{\varepsilon J \cos 2\phi}{2}\frac{\partial}{\partial J}(J) = -\frac{\varepsilon}{2}\cos 2\phi$$

The Linearisation is

$$\begin{pmatrix} \frac{\partial^2 K}{\partial J \partial \phi} & \frac{\partial^2 K}{\partial J^2} \\ -\frac{\partial^2 K}{\partial \phi^2} & -\frac{\partial^2 K}{\partial J \partial \phi} \end{pmatrix} = \begin{pmatrix} -\frac{\varepsilon}{2}\cos 2\phi & \frac{\varepsilon}{4} \\ \varepsilon J \sin 2\phi & \frac{\varepsilon}{2}\cos 2\phi \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{\varepsilon}{2}\cos(-\pi/2) & \frac{\varepsilon}{4} \\ \varepsilon J \sin(-\pi/2) & \frac{\varepsilon}{2}\cos\left(-\frac{\pi}{2}\right) \end{pmatrix} = \begin{pmatrix} 0 & \frac{\varepsilon}{4} \\ -\varepsilon J & 0 \end{pmatrix}$$

The Linearisation has determinant $\frac{\varepsilon^2}{4}J > 0 \therefore$ the fixed point is stable (a minimum)

Which is equivalent to your ~~[illegible]~~ approach above

Alternatively, expand the Hamiltonian around the fixed point

$$K = K(\phi_f, J_f) + \frac{\partial K}{\partial \phi}(\phi - \phi_f) + \frac{\partial K}{\partial J}(J - J_f)$$
$$+ \frac{1}{2}\frac{\partial^2 K}{\partial \phi^2}(\phi - \phi_f)^2 + \frac{1}{2}\frac{\partial^2 K}{\partial J^2}(J - J_f)^2 + \frac{\partial^2 K}{\partial \phi \partial K}(J - J_f)(\phi - \phi_f)$$