$I = \frac{2\sqrt{2m}\, E_1^{3/2}}{3\pi a} \Rightarrow E^{3/2} = \frac{3\pi a}{2\sqrt{2m}} I$

$\therefore E(I) = \left(\frac{3\pi a}{2\sqrt{2m}}\right)^{2/3} I^{2/3}$

$\omega(I) = \frac{\partial E(I)}{\partial I} = \left(\frac{3\pi a}{2\sqrt{2m}}\right)^{2/3} \frac{\partial}{\partial I}\left(I^{2/3}\right)$

$\omega(I) = \frac{2}{3}\left(\frac{3\pi a}{2\sqrt{2m}}\right) I^{-1/3}$ ✓

Put $A = \left(\frac{3\pi a}{2\sqrt{2m}}\right)^{2/3}$ ✓

then $E(I) = AI^{2/3}$, $\omega(I) = \frac{2}{3} A I^{-1/3}$ ✓

$\Theta = \int_0^q \frac{\partial p}{\partial I}\, dq$

$= \int_0^q \frac{\partial}{\partial I}\left(\sqrt{2m}\sqrt{AI^{2/3} - aq}\right) dq$

$= \sqrt{2m} \cdot \frac{A}{2} \cdot \frac{2}{3I^{1/3}} \int_0^q \frac{dq}{\sqrt{AI^{2/3} - aq}}$

$= \frac{A\sqrt{2m}}{3I^{1/3}} \int_0^q \frac{dq}{\sqrt{E - aq}}$

Sub $q = \frac{E}{a}\sin^2\phi$ $\quad q = 0 \Rightarrow \phi = 0 \quad q \neq 0 \Rightarrow \phi = \sin^{-1}\left(\sqrt{\frac{aq}{E}}\right)$

$dq = \frac{2E}{a}\sin\phi\cos\phi\, d\phi$

no need for trig substitution

$\Theta = \frac{A\sqrt{2m}}{3I^{1/3}} \int_0^\phi \frac{1}{\sqrt{E}\cos\phi} \cdot \frac{2E}{a}\sin\phi\cos\phi\, d\phi$

a simple algebraic one $u = E - aq$ would give result immediately

$= \frac{A\sqrt{2m}}{3I^{1/3}} \cdot \frac{2\sqrt{E}}{a} \int \sin\phi\, d\phi$

$= \frac{2\sqrt{2m}A\sqrt{E}}{3aI^{1/3}} \left[-\cos\phi\right]_0^\phi$

$q = \frac{E}{a}\sin^2\phi \Rightarrow \sin^2\phi = \frac{aq}{E}$

$-\cos\phi = -\sqrt{1 - \sin^2\phi} = -\sqrt{1 - \frac{aq}{E}}$

$\therefore \Theta = \frac{2\sqrt{2m}A\sqrt{E}}{3aI^{1/3}} \left[-\sqrt{1 - \frac{aq}{E}}\right]_0^q$

$= \frac{2\sqrt{2m}A\sqrt{E}}{3aI^{1/3}} \left(1 - \sqrt{1 - \frac{aq}{E}}\right)$

1 Long!

$E = AI^{2/3} = \left(\frac{3\pi a}{2\sqrt{2m}}\right)^{2/3} I^{2/3}$

$\therefore \sqrt{E} = \left(\frac{3\pi a}{2\sqrt{2m}}\right)^{1/3} I^{1/3}$