

Question 4

- (i) Sketch the graph of the function

$$V(q) = \cos 2q - 3 \cos q, \quad -\pi \leq q \leq \pi.$$

Explain why this function is suitable to act as a potential function for motion in a circle.

[6]

- (ii) A particle moves on a circle, with Hamiltonian

$$H(q, p) = \frac{1}{2}p^2 + V(q).$$

Using the sketch graph from part (i), draw a phase portrait for this system, being careful to point out all significant features such as fixed points and separatrixes, and explaining how you obtained your results.

[10]

- (iii) Let the particle's energy be E . Write down the ranges of values of E for which

- (a) no motion is possible,

[1]

- (b) the motion is librational,

[2]

- (c) the motion is rotational,

[1]

and give the values of E on any separatrixes.

[1]

- (iv) Express as an integral the period of

- (a) a rotation with energy E ,

[2]

- (b) the libration with energy E in the range $-\frac{17}{8} < E < -2$.

[2]