TMA MS323 02 **Cut-off date** 19 June 1996

(Covering *Units 5, 6* and *7* of Block II.)

Each question is marked out of 25. Your overall grade will be determined by the sum of your marks for all four questions.

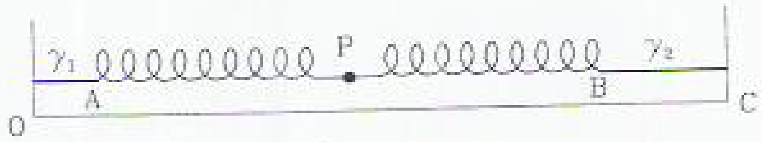
Question 1

(a) Find the Hamiltonian corresponding to the Lagrangian

$$L(q,u) = q^2\left(\tfrac{1}{2}u^2 - 1\right), \qquad q > 0.$$

Write down Lagrange's equation of motion for this Lagrangian, and confirm that it is equivalent to Hamilton's equations for the Hamiltonian you obtain. [8]

(b)



A particle P of mass m moves in a straight line on a smooth horizontal surface under the influence of two perfect springs AP and PB of natural length l_0 and stiffness k^2. The ends A and B of the springs are made to oscillate about the stationary points O and C respectively, such that $OA = \gamma_1(t)$ and $BC = \gamma_2(t)$, where γ_1 and γ_2 are known functions of time.

If the distance OC is d and the distance of the mass P from the oscillating point A is x, show that the Lagrangian function $L(x, \dot{x}, t)$ can be written as

$$L(x,\dot{x},t) = \tfrac{1}{2}m\dot{x}^2 - k^2x^2 - x\left\{k^2(\gamma_1 + \gamma_2 - d) + m\ddot{\gamma}_1\right\}.$$ [14]

[Hint: The force exerted by a spring of stiffness k^2 when its extension is e has magnitude k^2e.]

(c) Using the Lagrangian obtained in part (b), find the equation of motion for the mass in terms of x. [3]