①

TMA 01 Pt 2

2(a) i) $v(x) = x^3 + x^2 - 5x + 3$

Fixed points at $v(x) = 0$. If $x = -3$ is a fixed point then $v(-3) = 0$:

$v(-3) = (-3)^3 + (-3)^2 - 5(-3) + 3$

$= -27 + 9 + 15 + 3 = 0$

$\therefore x = -3$ is a fixed point and $(x - -3) = (x+3)$ is a factor of $x^3 + x^2 - 5x + 3$

$$\begin{array}{r|l} & x^2 - 2x + 1 \\ \hline x+3 & x^3 + x^2 - 5x + 3 \\ & x^3 + 3x^2 \\ \hline & \quad -2x^2 - 5x \\ & \quad -2x^2 - 6x \\ \hline & \qquad x + 3 \end{array}$$

✓

$\therefore v(x) = (x+3)(x^2 - 2x + 1)$

$v(x) = (x+3)(x-1)^2$ ✓

Check: $(x+3)(x-1)^2 = (x+3)(x^2 - 2x + 1)$

$= x^3 - 2x^2 + x + 3x^2 - 6x + 3$

$= x^3 + x^2 - 5x + 3$ ✓

There is only one other fixed point, at $x = 1$. The elementary invariant sets are those regions of phase space containing all the motion starting in them, which cannot be decomposed into smaller invariant sets ie for this system they are: $(-\infty, -3)$, $(-3, 1)$, $(1, \infty)$ together with the fixed points $x = -3, 1$. ✓

$\frac{7}{7}$