(b) We have

$$\sqrt{v(x)} = (1-x)\sqrt{x+3}, \quad -3 \le x \le 1,$$

which is zero at $x = -3$ and $x = 1$ and positive elsewhere (the choice of the factor $(1-x)$ being made to ensure this). We set $(1-x)\sqrt{x+3} = V(x)$, and consider the system with velocity function $V(x)$, for $-3 \le x \le 1$.

There are natural boundaries at $x = -3$ and 1. Between them, since the velocity function is everywhere positive, motion is from left to right.

Near $x = 1$ we have

$$V(x) \simeq 2(1-x), \qquad \text{for } x < 1,$$

so this is a simple stable fixed point which the phase points approach asymptotically as $t \to \infty$. (In fact, for $x_0 \simeq 1$ we have $x \simeq 1 - (1-x_0)\exp(-2(t-t_0))$.)

Near $x = -3$ we have

$$V(x) \simeq 4\sqrt{x+3}.$$

This is similar to the case discussed in Example 2.8, Unit 2, page 21: see Equation 2.16. By adapting that solution we see that, near -3,

$$x(t) \simeq (2t + \sqrt{x_0+3})^2 - 3.$$

The phase point which is at x_0 when $t = 0$ left the boundary point $x = -3$ a finite time beforehand.

(Note that this system can be integrated, though this is not necessary and the above qualitative discussion is all that is required to answer the question. We have

$$\dot{x} = (1-x)\sqrt{x+3} \quad \text{or}$$

$$t = \int_{x_0}^{x} \frac{dx}{(1-x)\sqrt{x+3}} = 2\int_{y_0}^{y} \frac{dy}{4-y^2}, \qquad y^2 = 3+x,$$

and integration gives

$$x(t) = 4\tanh^2(t/2 + A) - 3 \qquad A > 0, \quad \text{where} \quad x(0) = 4\tanh^2(A) - 3.$$

Thus $x(t) = -3$ at $t = -2A$ and $x(t) \to 1$ asymptotically as $t \to \infty$.)