(9)

$$v_1(x) = \frac{dx_1}{dt} = \sqrt{x+3}\left(4 - (x+3) + \underbrace{O((x+3)^2)}_{=0}\right)$$

$$\frac{dx_1}{dt} = 4\sqrt{x+3} + O((x+3)^{3/2})$$

$$\therefore \frac{dx_1}{dt} \approx 4\sqrt{x+3}$$

$$\int_{x_0}^{x} \frac{dx}{(x+3)^{1/2}} = \int_0^t 4\,dt$$

$$\left[2(x+3)^{1/2}\right]_{x_0}^{x} = 4t$$

$$2(x+3)^{1/2} - 2(x_0+3)^{1/2} = 4t$$

$$(x+3)^{1/2} = 2t + (x_0+3)^{1/2}$$

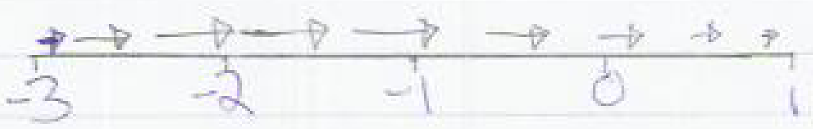
$$x+3 = \left(2t + (x_0+3)^{1/2}\right)^2$$

$$x(t) = \left(2t + (x_0+3)^{1/2}\right)^2 - 3$$

We are taking the +ve root of $(x_0+3)^{1/2}$, so $(2t + (x_0+3)^{1/2})^2 > 0$ for $t > 0$. And as t increases, $(2t + (x_0+3)^{1/2})^2$ increases, so x moves away from $x = -3$. The equation also tells us that the boundary point at $x=-3$ was left at a time $t < 0$ given by $t = \frac{-(x_0+3)^{1/2}}{2}$ (The equation breaks down for x equals -3: we can only actually say that x was closer to -3 for $t < 0$ for $x_0 > -3$)

We can see the logic of this from the phase diagram: $v_1(x) \geq 0$ for $-3 \leq x \leq 1$ $\therefore$ x is moving to the right for all $-3 < x < 1$ and t (since we've taken the +ve root for $v_1(x)$).

$\frac{9}{9}$



Total 25