(a2)

iii) $\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{(1+a\sin q)}$

$p = \sqrt{2(1+a\sin q)(E+\cos q)}$ from pt (ii)

$\therefore \dot{q} = \frac{\sqrt{2(1+a\sin q)(E+\cos q)}}{(1+a\sin q)}$

$\dot{q} = \frac{dq}{dt} = \sqrt{\frac{2(E+\cos q)}{(1+a\sin q)}}$

$dt = \sqrt{\frac{1+a\sin q}{2(E+\cos q)}}\,dq$

$\int_0^T dt = \int_{q_1}^{q_2} \sqrt{\frac{(1+a\sin q)}{2(E+\cos q)}}\,dq - \int_{q_2}^{q_1} \sqrt{\frac{1+a\sin q}{2(E+\cos q)}}\,dq$

$= 2\int_{q_1}^{q_2} \sqrt{\frac{1+a\sin q}{2(E+\cos q)}}\,dq$. ✓

q turning points are solns to $E+\cos q = 0$ ie $q = \pm\cos(-E)$ since cos is an even function. Put $\cos(-E) = \alpha$ (For some reason, the authors of the question have replaced q by x.) No idea why! Perhaps they thought it made the question easier!

$\therefore q_2 = \alpha$ and $q_1 = -\alpha$

$\therefore T = \int_0^T dt = 2\int_{-\alpha}^{\alpha} \sqrt{\frac{1+a\sin x}{2(E+\cos x)}}\,dx$

$T = \sqrt{2}\int_{-\alpha}^{\alpha} dx \sqrt{\frac{1+a\sin x}{E+\cos x}}$ as required ✓

Since this particular librational motion is about the centre, and $-1 < E < 1$, $\alpha = \cos(-E) \Rightarrow 0 \leq \alpha < \pi$ ✓

$\frac{6}{6}$

iv) $\cos x = 1 - \frac{x^2}{2!} + O(x^4)$

but $\cos\alpha = -E = -(-1+\delta^2/2) = 1 - \delta^2/2$

$\therefore \quad \delta \simeq \alpha$

$\alpha = O(\delta)$.