$H = \int y\,dy + \int (x + e^{2x})\,dx$

$= \frac{y^2}{2} + \frac{x^2}{2} + \frac{e^{2x}}{2}$

But $y = \dot{x}$

$\therefore H = \frac{\dot{x}^2}{2} + \frac{x^2}{2} + \frac{e^{2x}}{2} = \frac{1}{2}(\dot{x}^2 + x^2 + e^{2x})$

Since the Hamiltonian is conserved, twice the value of the Hamiltonian is conserved ✓

ie $\frac{1}{2}(\dot{x}^2 + x^2 + e^{2x}) = (\text{a constant}) = K$

So $\dot{x}^2 + x^2 + e^{2x} = 2K$ as required ✓

Check: $\frac{dH}{dt} = \frac{\partial H}{\partial x}\frac{dx}{dt} + \frac{\partial H}{\partial y}\frac{dy}{dt}$

$= -\dot{y}\dot{x} + \dot{x}\dot{y} \quad (=0)$

$\cancel{=} -(-x - e^{2x})y + y(-x - e^{2x})$

$= (x + e^{2x})y - (x + e^{2x})y$

$= 0$ as required.

To find the nature of the fixed point for the non linear system find the determinant of the matrix

$$\begin{pmatrix} \frac{\partial^2 H}{\partial x \partial y} & \frac{\partial^2 H}{\partial y^2} \\ \frac{\partial^2 H}{\partial x^2} & \frac{\partial^2 H}{\partial y \partial x} \end{pmatrix} = \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{pmatrix}$$

and apply the $AC - B^2$ test.

$$\det \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial x} \end{pmatrix} = \det \begin{pmatrix} 0 & 1 \\ -1-e^{2x} & 0 \end{pmatrix}$$

$= 1 + e^{2x} > 0$

So the fixed point is a centre ✓