(8)

Near $x=1$, $\sqrt{x+3}$ is well defined, so expand it in a Taylor series around $x=1$ and sub into $v_1(x)=(1-x)\sqrt{x+3}$ for $\sqrt{x+3}$

at $x=1$, $\sqrt{x+3} = \sqrt{1+3} = 2$

$\frac{d(\sqrt{x+3})}{dx} = \frac{1}{2\sqrt{x+3}}$; at $x=1$, $\frac{d(\sqrt{x+3})}{dx} = \frac{1}{2\sqrt{1+3}} = \frac{1}{4}$

So the Taylor series for $v_1(x)$ is

$$v_1(x) = (1-x)\left(2 + \frac{(x-1)}{4} + O((x-1)^2)\right)$$

$$v_1(x) = 2(1-x) + O((x-1)^2)$$

Ignore the $O((x-1)^2)$ term to give

$\frac{dx_1}{dt} = 2(1-x)$ (since $O((x-1)^2)$ is small compared to $2(1-x)$)

so $\int_{x_0}^{x} \frac{dx}{(1-x)} = \int_0^t 2\,dt$

$$\left[-\ln(1-x)\right]_{x_0}^{x} = 2t$$

$$-\ln\left(\frac{1-x}{1-x_0}\right) = 2t$$

$$(1-x) = (1-x_0)e^{-2t}$$

$$x(t) = 1-(1-x_0)e^{-2t}$$

Initially $x_0 < 1$ since $-3 \leq x \leq 1$.

at $t \to \infty$ $(1-x_0)e^{-2t} \to 0^+$ $\therefore$

$1-(1-x_0)e^{-2t} \to 1^-$, but since $(1-x_0)e^{-2t} \neq 0$ for any finite t, $x(t)$ does not reach $x=1$ in any finite time ✓

I find it easier to define a new variable So if $x \sim 1$ and $x<1$ then write $x = 1-\xi$ where $\xi > 0$ then $\dot{x} = -\dot{\xi}$ and $v(x) = (1-x)\sqrt{x+3}$ becomes $-\dot{\xi} = \xi\sqrt{4-\xi}$ $\sim 2\xi$ so $\dot{\xi} \sim -2\xi$ etc.

Seems easier to me!

Near $x=-3$, $(1-x)$ is well defined, so expand it in a Taylor series around $x=-3$, and sub into $v_1(x)$ for $(1-x)$.

at $x=-3$, $(1-x) = (1--3) = 4$

$\frac{d(1-x)}{dx} = -1$

or write $x = -3+\xi$

then $\dot{\xi} = (4-\xi)\sqrt{\xi}$ $\sim 4\sqrt{\xi}$