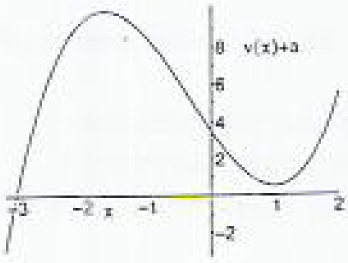

Figure 3 Graphs of the velocity function of Figure 1 plus a small constant.

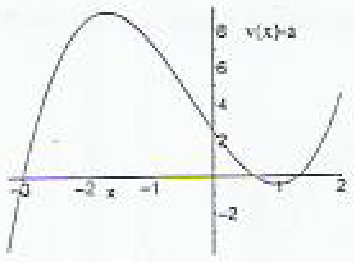

Figure 4 Graphs of the velocity function of Figure 1 minus a small constant.

In both cases the fixed point at $x = -3$, being simple, is not significantly affected by the change; it is moved slightly but its nature remains unchanged.

The non-simple fixed point at $x = 1$ is, however, changed dramatically, with the effect depending upon the sign of the perturbation. On the left hand side of the figure we show the graph of $v(x) + a$, $a > 0$. The non-simple fixed point at $x = 1$ has become merely a point at which the phase velocity has a local minimum. There is only one fixed point, which is unstable; it is obtained by moving the original unstable fixed point at $x = -3$ slightly. (The new position of the unstable fixed point is given by $x \simeq -3 - a/16$.)

On the right hand side of the figure we show the graph of $v(x) - a$, $a > 0$. In this case, the non-simple fixed point at $x = 1$ has become two simple fixed points, with the one corresponding to the smaller root being stable and the other unstable. The fixed point originally at $x = -3$ is just moved slightly, its nature being unchanged. (The fixed points are given approximately by $x \simeq 1 \pm \sqrt{a/4}$, $x \simeq -3 + a/16$.)

Thus the structure of the flow is changed significantly by the perturbation, the original two fixed points becoming either one or three.