$$\det\begin{pmatrix} 2x-\lambda & 0 \\ y & 2y+x-\lambda \end{pmatrix} = \det\begin{pmatrix} -4-\lambda & 0 \\ -1 & -2-2-\lambda \end{pmatrix}$$

$= (-4-\lambda)(-4-\lambda) = 0 \Rightarrow \lambda = -4$

Find the eigenvectors

$$\begin{pmatrix} -4--4 & 0 \\ -1 & -4--4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0.$$

$$\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ There is only one eigenvector, and the fixed point is stable since the single eigenvalue $\lambda<0$ $\therefore$ this fixed point is a strongly ~~unstable~~ stable improper node (Alternatively, $A \neq kI$ $\therefore$ improper node, $\lambda<0$ $\therefore$ strongly stable.)

$(-2,3)$

$$\det\begin{pmatrix} 2x-\lambda & 0 \\ y & 2y+x-\lambda \end{pmatrix} = \det\begin{pmatrix} -4-\lambda & 0 \\ 3 & 6-2-\lambda \end{pmatrix}$$

$$= \det\begin{pmatrix} -4-\lambda & 0 \\ 3 & 4-\lambda \end{pmatrix} = 0 \Rightarrow (-4-\lambda)(4-\lambda) = 0$$

$\therefore \lambda = -4, 4$. There are two eigenvalues with opposite signs $\therefore$ this fixed point is a saddle and not strongly stable or unstable

$(2,-3)$

$$\det\begin{pmatrix} 2x-\lambda & 0 \\ y & 2y+x-\lambda \end{pmatrix} = \det\begin{pmatrix} 4-\lambda & 0 \\ -3 & -6+2-\lambda \end{pmatrix}$$

$$= \det\begin{pmatrix} 4-\lambda & 0 \\ -3 & -4-\lambda \end{pmatrix} = 0 \Rightarrow (4-\lambda)(-4-\lambda) = 0$$

$\lambda = -4, 4$ $\therefore$ This fixed point is a saddle and not strongly stable or unstable

$(2,1)$

$$\det\begin{pmatrix} 2x-\lambda & 0 \\ y & 2y+x-\lambda \end{pmatrix} = \det\begin{pmatrix} 4-\lambda & 0 \\ 1 & 2+2-\lambda \end{pmatrix}$$

$$= \det\begin{pmatrix} 4-\lambda & 0 \\ 1 & 4-\lambda \end{pmatrix} = 0 \Rightarrow (4-\lambda)(4-\lambda) = 0$$

$\lambda = 4$