$\dot{q} = 0 \Rightarrow \dot{q} = \frac{p}{(1+a\sin q)} = 0 \Rightarrow p = 0$

$\dot{p} = 0 \Rightarrow \dot{p} = \frac{-a p^2 \cos q}{2(1+a\sin q)^2} + \sin q = 0$

But $p = 0 \therefore \dot{p} = \sin q = 0 \Rightarrow q = n\pi$

$n = 0, \pm 1, \pm 2$

The linearisation is given by

$$\begin{pmatrix} \frac{\partial^2 H}{\partial p \partial q} & \frac{\partial^2 H}{\partial p^2} \\ -\frac{\partial^2 H}{\partial q^2} & -\frac{\partial^2 H}{\partial p \partial q} \end{pmatrix}$$

$$\begin{pmatrix} \frac{-a p \cos q}{(1+a\sin q)^2} & \frac{1}{(1+a\sin q)} \\ -\left(\frac{a p^2 \sin q + a^2 p^2 \cos^2 q + a^2 p^2}{2(1+a\sin q)^3} + \cos q\right) & \frac{a p \cos q}{(1+a\sin q)^2} \end{pmatrix}$$

whose determinant is, at the fixed points (for which $p=0, q=n\pi$)

$$\det \begin{pmatrix} 0 & 1 \\ -\cos(n\pi) & 0 \end{pmatrix} = +\cos(n\pi) = +(-1)^n$$

For n odd the $AC-B^2$ test gives $AC-B^2 = -1 \therefore$ the fixed points at $((2n+1)\pi, 0)$ are saddles.

For n even the $AC-B^2$ test gives $AC-B^2 = 1 \therefore$ the fixed points at $q = 2n\pi$ are centres.

Also $A = \frac{\partial^2 H}{\partial q^2}(2\pi n, 0) = \cos 2\pi n = 1 > 0$

$\frac{4}{4}$

$\therefore$ The fixed points at $(2\pi n, 0)$ are minima.

ii) $H(q,p) = \frac{p^2}{2(1+a\sin q)} - \cos q$