(a-i) Substituting $x = -3$ into the velocity function gives $v(-3) = 0$, so the point $x = -3$ is a fixed point.

It follows that the velocity function can be written in the form

$$v(x) = (x+3)(x^2 + ax + 1) = x^3 + (a+3)x^2 + (3a+1)x + 3$$

for some constant a. Then by comparing the coefficient of x with that in the original expression we find that $a = -2$, so

$$v(x) = (x+3)(x^2 - 2x + 1) = (x+3)(x-1)^2.$$

Thus the fixed points are at $x = -3$ and $x = 1$.

The invariant sets are

$$(-\infty, -3), \quad x = -3, \quad (-3, 1), \quad x = 1, \quad (1, \infty).$$

A graph of this velocity function is shown in the figure, together with the phase diagram.

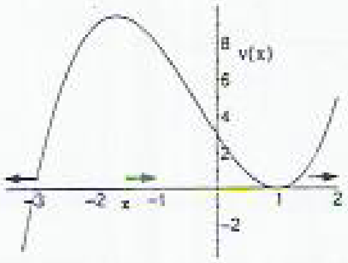

Figure 2 Graph of the velocity function $v(x) = (x+3)(x-1)^2$, with arrows showing the flow direction.

We have

$$v'(x) = (x-1)(3x+5),$$

and at $x = -3$, $v'(-3) = 16$, thus this is a simple fixed point and is unstable.

At $x = 1$ we have $v'(1) = 0$ so this fixed point is not simple: this is also obvious from the factorisation of $v(x)$, which shows that at $x = 1$ the velocity function has a double root. Since $v(x) > 0$ immediately adjacent to $x = 1$ phase points approach $x = 1$ from the left and recede from $x = 1$ on the right: that is, it is neither stable nor unstable.

(a-ii) Since $v(x) > 0$ for $x > 1$, it follows that for a phase point initially at $x = 2$, $x(t)$ is always increasing.

For large x, $v(x)$ behaves as x^3 so, on using the solution to Exercise 2.27, Subsection 2.4.1, page 20, it follows that the motion terminates.

(a-iii) Consider the cases of a positive and negative addition separately, as shown in the figure.