

3

Also, as  $x$  becomes larger the highest power of  $x$  becomes dominant, so for large  $x$ ,  $v(x) \approx x^3$ ,  $\frac{dv}{dx} \approx 3x^2$

so for a phase point initially at  $x=2$ ,  $x$  increases without bound.

$$\frac{dx}{dt} = v(x) = (x-1)^2(x+3)$$

dt

$$\therefore \int_0^t dt = \int_2^x \frac{dx}{(x-1)^2(x+3)}$$

$$\frac{1}{(x-1)^2(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+3)}$$

$$\therefore 1 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$$

oc=1

$$1 = A(1-1)(1+3) + B(1+3) + C(1-1)^2$$

$$1 = 4B \Rightarrow B = 1/4$$

oc=-3

$$1 = A(-3-1)(-3+3) + 1/4(-3+3) + C(-3-1)^2$$

$$1 = 16C \Rightarrow C = 1/16$$

oc=0

$$1 = A(0-1)(0+3) + 1/4(0+3) + 1/16(0-1)^2$$

$$= -3A + 3/4 + 1/16$$

$$-3A = 1 - 3/4 - 1/16 = (16 - 12 - 1)/16$$

$$-3A = 3/16 \Rightarrow A = -1/16$$

$$\therefore \frac{1}{(x-1)^2(x+3)} = \frac{-1/16}{(x-1)} + \frac{1/4}{(x-1)^2} + \frac{1/16}{(x+3)}$$

Check:

$$\begin{aligned} \frac{1}{(x-1)^2(x+3)} &= \frac{-1/16}{(x-1)} + \frac{1/4}{(x-1)^2} + \frac{1/16}{(x+3)} \\ &= \frac{-1/16(x-1)(x+3) + 1/4(x+3) + 1/16(x-1)^2}{(x-1)^2(x+3)} \end{aligned}$$

It would have been sufficient to evaluate the approximate relation

$$\dot{x} = v(x) \sim x^3$$

to show that minima is bounded

or quote some results.

Actually no analysis was required for full marks.

4/4