$\dot{x} = y \Rightarrow \frac{\partial \dot{x}}{\partial x} = \frac{\partial}{\partial x}(y) = 0$

$\frac{\partial \dot{x}}{\partial y} = \frac{\partial}{\partial y}(y) = 1$

$\dot{y} = -x - e^{2x} \Rightarrow \frac{\partial (\dot{y})}{\partial x} = \frac{\partial}{\partial x}(-x - e^{2x}) = -1 - 2e^{2x}$

$\frac{\partial}{\partial y}(\dot{y}) = \frac{\partial}{\partial y}(-x - e^{2x}) = 0.$

The eigenvalues of the Jacobian matrix give us the nature of the fixed point for the linearisation, and are found from

$$\det\begin{pmatrix} \frac{\partial \dot{x}}{\partial x} - \lambda & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} - \lambda \end{pmatrix} = 0$$ Looks complicated.

all you need is that this is >0. which it is for all x.

$$= \det\begin{pmatrix} 0 - \lambda & 1 \\ -1 - 2e^{2x} & 0 - \lambda \end{pmatrix} = \lambda^2 + (1 + 2e^{2x}) = 0$$ where does this come from.

$\therefore \lambda = \sqrt{-1 - 2e^{2x}} \; (= \sqrt{-1 - 2e^{-0.35x}} = \pm 1.36i)$ I now see

Since $e^{2x} > 0$, $-1 - 2e^{2x} < 0 \therefore$ the eigenvalues are complex, and the fixed point of the linearised system is a centre.
(Also see opposite)

You notation leaves a bit to be desired

$\frac{\partial \dot{x}}{\partial x} = -\frac{\partial \dot{y}}{\partial y} = 0 \therefore$ The system is Hamiltonian.

Probably better to use u for $\dot{x}$, v for $\dot{y}$ as in text then easier to follow.

Since $\dot{x} = \frac{\partial H}{\partial y}$ and $\dot{y} = -\frac{\partial H}{\partial x}$

The Hamiltonian is given by

$H = \int \dot{x}\,dy - \int \dot{y}\,dx$

$= \int y\,dy - \int (-x - e^{2x})\,dx$