

$A = \frac{\partial^2 H}{\partial x^2} = 1 + 2e^{2x} > 0$. The fixed point is a minimum. ✓

(7/7)

b) $\dot{x} = x^2 - 4$, $\dot{y} = y^2 + xy - 3$
fixed points are at $\dot{x} = \dot{y} = 0$
i.e. $0 = x^2 - 4 = (x+2)(x-2) \Rightarrow x = -2, 2$
 $0 = y^2 + xy - 3$
Put $x = -2$

$0 = y^2 - 2y - 3 = (y+1)(y-3) \Rightarrow y = -1, 3$
 $\therefore (-2, -1), (-2, 3)$ are fixed points

Put $x = 2$

$0 = y^2 + 2y - 3 = (y+3)(y-1) \Rightarrow y = -3, 1$
 $\therefore (2, -3), (2, 1)$ are fixed points.

Nature of the fixed points is given by the eigenvalues of the Jacobian matrix.

Noting again.

$\frac{\partial \dot{x}}{\partial x} = \frac{\partial}{\partial x}(x^2 - 4) = 2x$

$\frac{\partial \dot{x}}{\partial y} = \frac{\partial}{\partial y}(x^2 - 4) = 0$

$\frac{\partial \dot{y}}{\partial x} = \frac{\partial}{\partial x}(y^2 + xy - 3) = y$

$\frac{\partial \dot{y}}{\partial y} = \frac{\partial}{\partial y}(y^2 + xy - 3) = 2y + x$

i.e. Jacobian matrix is

$\begin{pmatrix} 2x & 0 \\ y & 2y+x \end{pmatrix}$, and its eigen-

values are found from

$\det \begin{pmatrix} 2x-1 & 0 \\ y & 2y+x \end{pmatrix} = 0$ for each fixed

point (x_f, y_f) .

$(-2, -1)$