3a) $\dot{x} + x + e^{2x} = 0$

Put $\dot{x} = y$, then equation becomes the system $\dot{x} = y$ ✓

$\dot{y} = -x - e^{2x}$ ✓

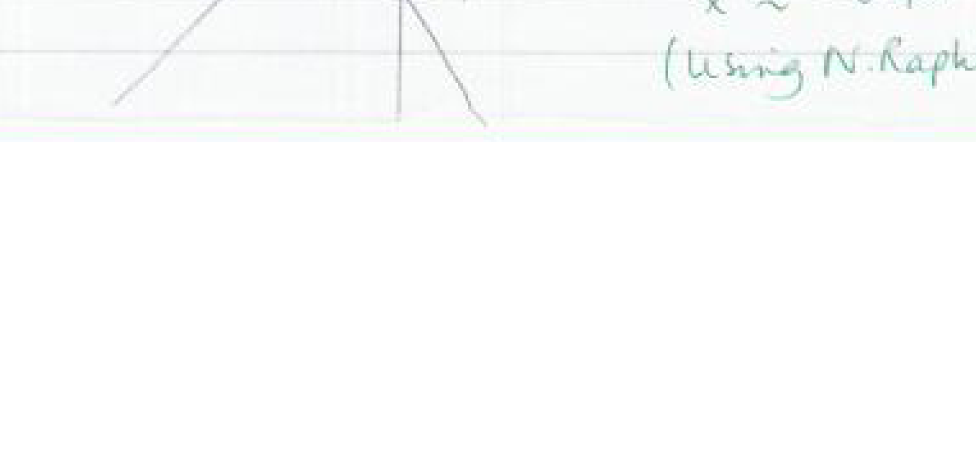
At the fixed points $\dot{x} = \dot{y} = 0$

$\dot{x} = 0 \Rightarrow \dot{x} = y = 0 \Rightarrow y = 0$

$\dot{y} = 0 \Rightarrow \dot{y} = -x - e^{2x} = 0 \Rightarrow x = -e^{2x}$

Put $g(x) = x$, $f(x) = -e^{2x}$

Then $g(x), f(x)$ are continuous $g(x)$ increases with increasing x $\left(\frac{dg(x)}{dx} = 1 > 0\right)$ and $f(x)$ decreases with increasing x $\left(\frac{df(x)}{dx} = -2e^{2x} < 0\right)$. The domain of both is $\mathbb{R}$, and since $f(-1) = -0.1353$, $f(0) = -1$ and $g(-1) = -1$, $g(0) = 0$, by applying the intermediate value theorem, we see that $g(x)$ and $f(x)$ intersection at least one point. Since $f(x)$ is decreasing, and $g(x)$ is increasing, this point of intersection is unique, hence there is only one $x \in \mathbb{R}$ for which $x = -e^{2x}$, so only one $x \in \mathbb{R}$ for which $\dot{y} = -x - e^{2x} = 0$ hence there is only one fixed point.



Yes.

Actually $x \approx -0.43$ (using N. Raphson)