$$\sinh(x-a)x = \varepsilon e^{-x}$$

$$\frac{1}{2}\left(e^{(x-a)} - e^{-(x-a)}\right) = \varepsilon e^{-x}$$

$$e^{x-a} - e^{-(x-a)} = 2\varepsilon e^{-x}$$

$$e^{2x-a} - e^{a} = 2\varepsilon$$

$$e^{2x-a} = 2\varepsilon + e^{a}$$

$$2x - a = \ln(2\varepsilon + e^{a})$$

$$x = \frac{a}{2} + \frac{1\ln(2\varepsilon + e^{a})}{2} \quad \checkmark$$

$$\frac{dx}{d\varepsilon} = \frac{d(a/2)}{d\varepsilon} + \frac{d}{d\varepsilon}\left(\tfrac{1}{2}\ln(2\varepsilon + e^{a})\right) \qquad 1$$

$$= \frac{1}{2\varepsilon + e^{a}}$$

$$\frac{d^2x}{d\varepsilon^2} = \frac{d}{d\varepsilon}\left(\frac{1}{2\varepsilon + e^{a}}\right) = \frac{-2}{(2\varepsilon + e^{a})^2} \qquad 1$$

$$\frac{d^3x}{d\varepsilon^3} = \frac{d}{d\varepsilon}\left(\frac{-2}{(2\varepsilon + e^{a})^2}\right) = \frac{8}{(2\varepsilon + e^{a})^3} \qquad 1$$

At $\varepsilon = 0$

$$\sinh(x-a) = 0 \Rightarrow x = a$$

$$\frac{dx(0)}{d\varepsilon} = \frac{1}{2\times 0 + e^{a}} = e^{-a} \quad \checkmark$$

$$\frac{d^2x(0)}{d\varepsilon^2} = \frac{-2}{(2\times 0 + e^{a})^2} = -2e^{-2a}$$

$$\frac{d^3x(0)}{d\varepsilon^3} = \frac{8}{(2\times 0 + e^{a})^3} = 8e^{-3a} \qquad 3$$

Then around $\varepsilon = 0$ the Taylor series approximating x is given by