

TMA 01 Part 1

Please ① Draw margins on right and left.

② Use ink not pencil

1) a) i) $u = \frac{1}{2} x^2$

$v = y^2/x$

$\frac{\partial u}{\partial x} = x$

$\frac{\partial v}{\partial y} = \frac{2y}{x}$

$\frac{\partial u}{\partial y} = 0$

$\frac{\partial v}{\partial x} = y^2 \frac{\partial}{\partial x} \left(\frac{1}{x} \right) = -\frac{y^2}{x^2}$

∴ Jacobian matrix is

$$\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} x & 0 \\ -\frac{y^2}{x^2} & \frac{2y}{x} \end{pmatrix}$$

Total $\frac{22}{25}$

$$\det \begin{pmatrix} x & 0 \\ -\frac{y^2}{x^2} & \frac{2y}{x} \end{pmatrix} = 2y$$



which is not area preserving in general

ii) $u = \tan x$

$v = (y-k)\cos^2 x$

$\frac{\partial u}{\partial x} = \sec^2 x$

$\frac{\partial v}{\partial x} = (y-k) \times -2 \sin x \cos x$

$\frac{\partial u}{\partial y} = 0$

$\frac{\partial v}{\partial y} = \cos^2 x$

Jacobian matrix is

$$\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} \sec^2 x & 0 \\ -2(y-k)\sin x \cos x & \cos^2 x \end{pmatrix}$$

$$\det \begin{pmatrix} \sec^2 x & 0 \\ -2(y-k)\sin x \cos x & \cos^2 x \end{pmatrix} = \sec^2 x \cos^2 x = 1$$

