ii) $f(x,y) = \sinh(x-a) - ye^{-x}$

$\dfrac{\partial f}{\partial x} = \cosh(x-a) + ye^{-x}$

$\dfrac{\partial f}{\partial y} = -e^{-x}$

$\dfrac{\partial^2 f}{\partial x \partial y} = e^{-x} = \dfrac{\partial^2 f}{\partial y \partial x}$ ✓

The Taylor expansion about $(a,0)$ is given by:

$$f(x,y) = f(a,0) + \frac{\partial f}{\partial x}(a,0)(x-a) + \frac{\partial f}{\partial y}(a,0)\,y + \frac{\partial^2 f}{\partial y \partial x}(a,0)(x-a)y + \cdots\cdots$$ ✓

$= \sinh(a-a) - 0 \times e^{-a} + (\cosh(a-a) + 0 \times e^{-a})(x-a) + -e^{-a}y + e^{-a}(x-a)y$

$= 0 - 0 + (x-a) - e^{-a}y + e^{-a}(x-a)$ ✓

$= (x-a) - e^{-a}y + e^{-a}(x-a)y$ as required

$\dfrac{6}{6}$

When $f(x,y) = 0$

$0 \approx (x-a) - e^{-a}y + e^{-a}y(x-a)$

$= (x-a)(1+e^{-a}) - e^{-a}y$

$(x-a)(1+e^{-a}) = e^{-a}y$

$x - a = \dfrac{ye^{-a}}{1+e^{-a}y} \Rightarrow x \simeq a + \dfrac{ye^{-a}}{1+e^{-a}y}$

3

$\dfrac{2}{5}$

Expand this as follows

$ye^{-a}(1+e^{-a})^{-1} = ye^{-a}(1 - ye^{-a})$ + higher order terms

$= ye^{-a} - y^2e^{-2a} + \cdots$