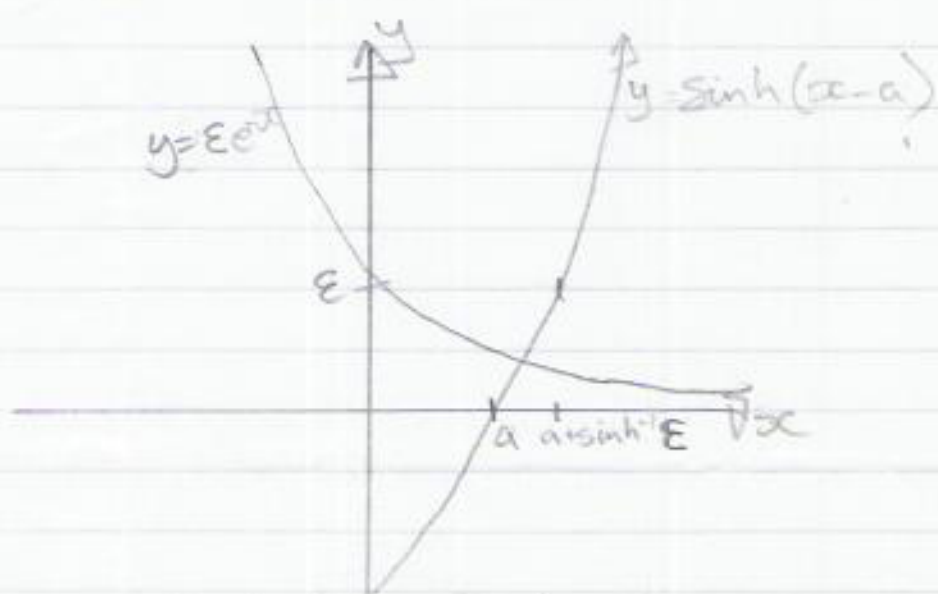


Drawn on the same axes they must intersect in the region for which $0 < x < \infty$.



$y = \sinh(x-a)$ is increasing with increasing x ($\frac{dy}{dx} = \cosh(x-a) > 0$)

It is continuous and passes through $(a, 0)$ and $(a + \sinh^{-1} \epsilon, 1)$.

$y = e^{-x}$ is decreasing ($\frac{dy}{dx} = -e^{-x} < 0$)

It passes through points $(0, \epsilon)$ and as $x \rightarrow \infty$, $y \rightarrow 0$ hence $y=0$ is an asymptote. By (a very slight adaptation of) the intermediate value theorem, there is a root $a > 0$ such that $\sinh(x-a) = \epsilon e^{-x}$

If $\epsilon = 0$ then $\sinh(x-a) = 0 \therefore x = a$.