(5)

$$\dim_H F = 1 - \frac{\ln 3}{\ln \lambda}$$ ✓

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b) Let $(x_1, y_1) = (x_{i_1 i_2 i_3 \ldots i_k i_{k+1} \ldots}, y_1)$
$(x_2, y_2) = (x_{i'_1 i'_2 i'_3 \ldots i'_k j_{k+1} \ldots}, y_2)$
with $i_n = i'_n$ for $n = 1$ to k but $i_{k+1} \neq j_{k+1}$, where the sequence $i_1 i_2 \ldots$ is written in base 3.
$|x_1 - x_2| \leq 3^{-k}$ but
$|f^{[k]}(x_1) - f^{[k]}(x_2)| \geq 1/3$ ✓
So no matter how close x_1, x_2 are originally if they are not identical, there will be an iteration for which they are far apart. ✓

The y coordinates are the same so that if (x_1, y) belongs to F, so too does (x_2, y)

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3)a) If $f(x) = x^2 + c$, then since $f^n(x) = f^{n-1} f(x)$ we can write $f^n(x) = x_n$ so $x_{n+1} = x_n^2 + c$ ✓
If c is real and $c > 1/4$, for some $\varepsilon > 0$, $x_{n+1} = x_n^2 + 1/4 + \varepsilon$ ✓

so $\begin{cases} x_{n+1} - x_n = x_n^2 - x_n + 1/4 + \varepsilon \\ \qquad = (x_n - 1/2)^2 + \varepsilon \geq \varepsilon \end{cases}$ ✓

so $\begin{cases} x_1 - x_0 \geq \varepsilon \\ x_2 - x_1 \geq \varepsilon \Rightarrow x_2 - x_0 \geq 2\varepsilon \\ x_3 - x_2 \geq \varepsilon \Rightarrow x_3 - x_0 \geq 3\varepsilon \end{cases}$ ✓

Stick to the notation with $F(x)$

$F^n(x)$! $F^n(x)$

$x_n - x_0 \geq n\varepsilon \Rightarrow x_n \to \infty$ as $n \to \infty$.

6 $\Rightarrow$ $f^k(0)$ not bounded as $k \to \infty$ ✓
$\Rightarrow$ $C \notin M$.