(2)

$S_3(x) = \frac{x}{16} + \frac{5}{8}$ ✓

S_1, S_2, S_3 are all contractions since $|S_i(x) - S_i(y)| = |x-y|/16$, $i = 1,2,3$ $x, y \in [0,1]$. It follows from Th 9.1 of Falconer that if $S = S_1 \cup S_2 \cup S_3$ then there is a unique non empty compact set F satisfying $F = S(F) = \bigcap_{k=1}^{\infty} S^k(E)$. F is invariant for f since $f(F) = f(S_1(F)) \cup f(S_2(F)) \cup f(S_3(F)) = F \cup F \cup F = F$ (where E is some interval). ✓

If $x < 0$ then $f(x) = 16x$ so $f^k(x) \to -\infty$ as $k \to \infty$. If $x > 3/4$ then $f(x) > 2x$ ✓ so $f^k(x) \to \infty$ as $k \to \infty$, ✓ so $F \subset [0, 0.75]$ ✓ *

Define $S = S_1 \cup S_2 \cup S_3$. If $x \in [0, 0.75] \setminus F$ then, for some k, $x \notin S^k([0, 0.75])$ ✓ so $f^k(x) \notin [0, 0.75]$ ✓ and $f^k(x) \to \pm\infty$ as $k \to \infty$, so F is indeed a repeller for f. (For defn of F see overleaf) ✓

$\frac{10\frac{1}{2}}{11}$

* 0.75 is not exact whereas 3/4 is.

c) Take V to be the open interval $(0,1)$

$S_i(V)$ are also all contained in V, so they satisfy the open set condition (9.11) of Falconer

$S_1(0,1) = (\cancel{2/16}\ 1/8, 3/16)$ ✓
$S_2(0,1) = (5/16, \cancel{6/16}\ 3/16)$ endpoints of interval in wrong order
$S_3(0,1) = (\cancel{10/16}\ 5/16, 11/16)$ ✓

and these are all disjoint. ✓
Each similarity is a scaling by 1/16. We solve for s where $s = \dim_H F$ from Theorem 9.3 of Falconer.

$$3 \cdot (1/16)^s = 1 \checkmark$$
$$\Rightarrow (1/16)^s = 1/3$$
$$-s \ln 16 = -\ln 3$$