3

$\Rightarrow s = \frac{\ln 3}{\ln 16}$ ✓, using Th. 9.3 ✓

$\frac{4\frac{1}{2}}{6}$

d) Let $i_1 i_2 i_3 \dots$ be an infinite sequence which contains every finite sequence of ones, twos and threes appearing as a consecutive block of terms.

If $x_{j_1 j_2 \dots} = \bigcap_{k=1}^{\infty} S_{j_1 j_2 \dots j_k}$ ... $[0, 0.~~75~~ \tfrac{3}{4}]$ ✓ is a point of F

then $f(x_{j_1 j_2 \dots}) = x_{j_2 j_3 \dots}$ ✓

Now if $x_{i_1' i_2' \dots} \in F$ and $q \in N$ ✓ then there exists $k \in N$ ✓ such that $(i_{k+1}, \dots, i_{k+q}) = (i_1', i_2', \dots, i_q')$ ✓ hence $|f^k(x_{i_1, i_2, \dots}) - x_{i_1', i_2', \dots}| \le 16^{-q}$ ✓

So there exists $x \in F$ such that $\{f^k(x)\}$ is dense in F. ✓

$\frac{5\frac{1}{2}}{6}$

e) The periodic points are dense in F since if $x_{j_1, j_2, \dots} \in F$ and $k \in N$ then the periodic point $x = x_{i_1, i_2, \dots i_k, i_1, \dots i_k, i_1 \dots}$ satisfies $i_n = j_n, n = 1$ to k, $|x_{i_1, i_2, \dots} - x_{i_1, i_2, \dots i_k, i_1, \dots i_k, i_1}| \le 16^{-k}$ and x is periodic with period k. ✓

$\frac{3}{3}$

$\frac{28}{30}$

Let $E_k = F^k(E)$

2) a) $f^k(E)$ consists of 3^k horizontal strips height λ^k and length 1, separated by gaps of at least $\lambda^{k-1}(1/3 - \lambda)$ since $f(f^k(E)) = f^{k+1}(E)$, ✓ and $f^k(E)$ E_k is a decreasing sequence of sets. The compact limit set $F = \bigcap_{k=1}^{\infty} f^k(E)$ satisfies $f(F) = F$ ✓. If $(x, y) \in E$ then $f^k(x, y) \in f^k(E)$ ✓ so

State also that F is made up of horizontal lines with at least 1 line in each strip of E_k