9

d) $(f^{-1})'(z) = \frac{1}{4}(z-c)^{-3/4}$ ✓ for $z \in A$

The ~~open~~ set A $\cancel{V} = \{z : |z| \le |2c|^{1/4}\}$
Denote the four branches of $f^{-1}(z)$ by $S_1(z), S_2(z), S_3(z), S_4(z)$ — Should have come much earlier
then $\bigcup_{i=1}^{4} S_i(\cancel{V}A) \subseteq \cancel{V}A$ from 4)a)

Since this is true for each i
Also the interior of each of the four loops is mapped bijectively on to $\cancel{V}A$, so each $S_i(\cancel{V}A)$ is the interior of one of the loops of $f^{-1}(C)$ ✓, where C is the circle $|z| = |2c|^{1/4}$, and these interior open sets are disjoint ✓. Note that $\cancel{V}A$ has been chosen so that $f^{-1}(\cancel{V}A) \subseteq \cancel{V}A$, but 'only just'. The "only just" applies to $F^{-1}(C)$

$$(f^{-1})'(z) = \frac{1}{4}(z-c)^{-3/4} \checkmark$$
so for $|z| = |2c|^{1/4}$ ✓

$$\frac{1}{4(|c|+|2c|^{1/4})^{3/4}} \le |(f^{-1})'(z)| = |S_i'(z)| \le \frac{1}{4(|c|-|2c|^{1/4})^{3/4}} \checkmark \text{ (A)}$$

This is a contraction (we know already from a)) if What is ?
$$\frac{1}{4(|c|-|2c|^{1/4})^{3/4}} < 1$$
ie $(|c|-|2c|^{1/4})^{3/4} > \frac{1}{4}$

$$|c| - |2c|^{1/4} > \frac{1}{4\sqrt[3]{4}}$$
$$|c|\left(1 - \left(\frac{2}{|c|^3}\right)^{1/4}\right) > \frac{1}{4\sqrt[3]{4}}$$

P.T.O.