4

$f^k(x,y)$ lies within a distance λ^k of F so all points of E are attracted to F ✓

$\frac{4}{5}$

Consider first the cantor set E constructed by the transformation

$$g(y) = \begin{cases} \lambda y & y \in [0, 1/3] \\ \lambda y + 1/3 & y \in (1/3, 2/3) \\ \lambda y + 2/3 & y \in (2/3, 1] \end{cases}$$

$S_1(y)$, $S_2(y)$, $S_3(y)$ ✓ ✓ ✓ $y \in [0,1]$ in all cases

At the stage k of construction we have 3^k intervals of length λ^k, which can be covered by such intervals

$$\Rightarrow \dim_H \cancel{E}\, F_1 \leq \dim_B \cancel{E}\, F_1 \leq \lim_{k \to \infty} \frac{\ln(3^k)}{-\ln(\lambda^k)}$$

$$= \frac{\ln 3}{-\ln \lambda}$$ ✓

On the other hand, let each of the 3^k intervals carry a mass of 3^{-k}. Let U be a set satisfying $\lambda^k(1/3 - \lambda) \leq |U| < \lambda^{k-1}(1/3 - \lambda)$ then U satisfies

$$\left(\frac{1/3 - \lambda}{\lambda}\right) \lambda^{k+1} \leq |U| \leq \lambda^k \cdot \frac{(1/3 - \lambda)}{\lambda}$$ ✓

$$\lambda^{k+1} \leq \frac{\lambda |U|}{(1/3 - \lambda)} \leq \lambda^k$$

* It is much easier to do this using Theorem 9.3 of Falconer

Since U intersects at most one interval

$$\mu(U) \leq 3^{-k} = 3 \cdot 3^{-(k+1)}$$

$$= 3\, \lambda^{(k+1)\ln 3/-\ln \lambda}$$

$$\leq 3 \left(\frac{\lambda}{1/3 - \lambda}\right)^{\ln 3/-\ln \lambda} |U|^{\ln 3/-\ln \lambda}$$

$$\Rightarrow \dim_H \cancel{E}\, F_1 \geq \frac{\ln 3}{-\ln \lambda}$$ ✓

$F = \cancel{E}\, F_1 \times [0,1]$ so by corollary 7.4 ✓