Question 4 – 30 marks

For $1 \le i \le 3$, let $S_i : \mathbf{R} \to \mathbf{R}$ be the affine transformation defined by

$$S_i(t, x) = (t/3 + (i-1)/3, a_i t + x/2 + b_i).$$

Let F be the invariant set of S_1, S_2 and S_3.

(a) Determine values of a_i and b_i $(1 \le i \le 3)$ for which F is a self-affine curve passing through the points $(0,0)$, $(1/3, 2)$, $(2/3, 1)$ and $(1, 0)$ with box dimension equal to $2 - \log 2/\log 3$.

(b) Let $E_0 = \{(x, y) : 0 \le x \le 1, y = 0\}$ and $E_{k+1} = \bigcup_{i=1}^{3} S_i(E_k)$, for each $k \in N$. Sketch the first two stages, E_1 and E_2, in the construction of F, using the values of a_i and b_i $(1 \le i \le 3)$ determined in part (a).

TMA M835 04 — Cut-off date 12 September 2001

Please make sure that the assignment number is correctly entered on your PT3 form as

$\boxed{\text{M835 04}}$.

This assignment assesses the material covered in Chapters 7 and 8 of the Course Notes.

Question 1 – 30 marks

Let $f : \mathbf{R} \to \mathbf{R}$ be the function defined by

$$f(x) = \begin{cases} 16x - 2 & x \le 1/4 \\ -16x + 6 & 1/4 < x < 1/2 \\ 16x - 10 & x \ge 1/2. \end{cases}$$

(a) Sketch the graph of f, identifying the values of x for which $f(x) = 0$ and any local maxima or minima of f.

(b) Find a repeller F for f.

(c) Determine $\dim_H F$.

(d) Show that there exists a point $x \in F$ such that the orbit $\{f^k(x)\}$ is dense in F.

(e) Show that the periodic points of f are dense in F.

Question 2 – 20 marks

Let $E = [0,1] \times [0,1]$ be the unit square and let $f : E \to E$ be the function defined by

$$f(x) = \begin{cases} (3x, \lambda y) & 0 \le x \le 1/3 \\ (3x - 1, \lambda y + 1/3) & 1/3 < x \le 2/3 \\ (3x - 2, \lambda y + 2/3) & 2/3 < x \le 1, \end{cases}$$

where $0 < \lambda < 1/3$.

(a) Show that there is a set F that is an attractor for f and determine $\dim_H F$.

(b) Show that f has sensitive dependence on initial conditions.