In this question, you should start by defining S_i for $i = 1$ to 4 to be the 4 branches of F^{-1}, where $S_i(D)$ are the interiors of the 4 loops of the curve $F^{-1}(C)$.

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4) a) As θ ranges over $[0, 8\pi]$

$|f^{-1}|$ ranges over

$$[(|c| - |2c|^{1/4})^{1/4}, (|c| + |2c|^{1/4})^{1/4}]$$

Since $|c| > 2$

$$|2c|^{1/4} < |c| \quad \checkmark \quad \text{so } A \subset D$$

$$\Rightarrow |c| + |2c|^{1/4} < 2|c|$$

$$\Rightarrow (|2c|^{1/4} + |c|)^{1/4} < |2c|^{1/4}$$

on $z = |2c|^{1/4}$ this says

$$|f^{-1}(z)| < z. \quad F^{-1} \text{ cannot be used as it has 4 branches}$$

$\frac{2}{7}$ So ~~f^{-1}~~ S_i ~~is a~~ are contractions, and S_i, $i = 1$ to 4

~~f^{-1}~~ maps A into A since f_c maps interiors of loops of f^{-1} bijectively onto D. *

b) The S_i + are contractions, so by + not defined

Th. 9.1 there is a unique non empty compact set F satisfying

$F = \bigcup_{i=1}^{4} S_i(F)$, with the $S_i(F)$ disjoint. ✓

A ~~V~~ contains at least one point z of J (for example a repelling periodic point) we have This has to be shown. See Course Notes, p. 160, remark 8.

$J = $ ** $\overline{\bigcup_{i=1}^{\infty} f^{-k}(z)} \subset \overline{V}$ A and J satisfies

** closure of since $F^{-k}(\;) \subset$

$\frac{4}{7}$ $J = f^{-1}(J) = \bigcup_{i=1}^{4} S_i(J)$. Thus $J = F$ ✓

since F is unique.

* You need to use the Complex Mean Value Theorem (which you do on the next page) to show that the S_i are contractions on A.