Please make sure that the assignment number is correctly entered on your PT3 form as

$$\boxed{\text{M835 03}}.$$

This assignment assesses the material covered in Chapters 5 and 6 of the Course Notes.

Question 1 – 20 marks

Let F be the set with generator E_1, as shown below, where the long line segments are $\{(x,y) : 0 \le x \le 1/3, y = 0\}$, $\{(x,y) : 1/3 \le x \le 2/3, y = 1/9\}$ and $\{(x,y) : 2/3 \le x \le 1, y = 0\}$,

and with E_2 as shown in the following picture.

(a) Write down similarities S_i, $1 \le i \le 5$, on $\mathbf{R}^2$ such that $E_2 = \bigcup_{i=1}^{5} S_i(E_1)$.

(b) Use Theorem 9.3 of Falconer to determine the Hausdorff and box dimensions of F.

Question 2 – 30 marks

Let F be the invariant set of the following three contractions on $[1,4]$:

$$S_1(x) = 1 + 1/x, \quad S_2(x) = 2 + 1/x \quad \text{and} \quad S_3(x) = 3 + 1/x.$$

(a) Use Proposition 9.7 of Falconer to show that $0.53 < \dim_H F \le 1$.

(b) Show that F is the set of positive real numbers with infinite continued fraction expressions which have partial quotients containing only the numbers 1, 2 and 3.

Question 3 – 20 marks

Let $f : [0,1] \to \mathbf{R}$ be defined by

$$f(t) = \sum_{k=1}^{\infty} \lambda^{(s-2)k} \sin(\lambda^k t + \theta_k),$$

where $\lambda > 1$, $1 < s < 2$ and $\theta_k \in \mathbf{R}$. Show that if λ is sufficiently large, then $\dim_B \text{graph } f = s$.