

(6)

If  $A$  is any subset of  $\mathbb{R}^*$  then we set  
 $\mu(A) = \inf \left( \sum \mu(U_i) : A \subset \bigcup U_i \text{ and } U_i \in \mathcal{E} \right)$  ✓

$\frac{5}{8}$  This construction follows that on p13-14 of Falconer. It follows from Prop 1.7 that this defines a measure  $\mu$  with support contained in  $F$ .

Explain your method by reference to Falconer (i) p 41.

Let  $E_k$  denote stage  $k$  in construction of  $F$ .  
For each interval in  $E_k$  take a closed disc diameter  $10^{-k}$  centred at the midpoint of the interval. There are  $5^k$  disc intervals in  $E_k$  and they cover  $F$  so

$$\dim_B F \leq \lim_{k \rightarrow \infty} \frac{\log 5^k}{-\log(10^{-k})} = \frac{\log 5}{\log 10}$$

\* actually only right hand endpoints are in  $F$

The  $5^k$  intervals that make up  $E_k$  have  $2 \times 5^k$  distinct endpoints ✓ each belonging to  $F$ . Balls radius  $\frac{1}{3} \times 10^{-k}$  (intervals in  $\mathbb{R}$ ) centred at the endpoints are mutually disjoint ✓

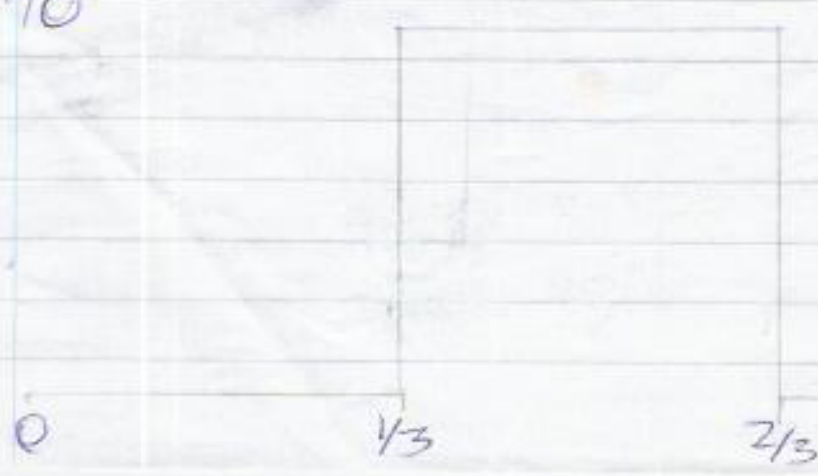
$$\dim_B F \geq \lim_{k \rightarrow \infty} \frac{\log(2 \times 5^k)}{-\log(\frac{1}{3} \times 10^{-k})}$$

(iv) Falconer p 41

$$\frac{5}{7} = \lim_{k \rightarrow \infty} \frac{\log 2 + k \log 5}{\log 3 + k \log 10} = \frac{\log 5}{\log 10}$$
$$\Rightarrow \underline{\dim_B F} = \overline{\dim_B F} = \frac{\log 5}{\log 10} \quad \checkmark$$

$\frac{13}{20}$

4) 0



$E_0$  ✓

$E_1$  ✓