Usually, one would put $\frac{1}{2^{k+1}} \leq \delta < \frac{1}{2^k}$ (9)

5) If $|U| = \delta < 1/2$ and k is the integer satisfying $1/2^{k+2} \leq \delta < 1/2^{k+1}$ then U can cover at most one of the points in the set $\{1/2, 1/4, \dots 1/2^k\}$

For your δ, you need $\frac{1}{2^{k+1}}$

Thus at least $k+1$ sets of diameter at most $|U|$ are required to cover this set and

lower dimension here

$$\dim_B F \geq \lim_{k\to\infty} \frac{\log(k+1)}{-\log(1/2^{k+2})} = \lim_{k\to\infty} \frac{\log k+1}{(k+2)\log 2} = 0$$

If $0 < \delta \leq 1/2$? No then take k such that $2^{-(k+1)} \leq \delta < 2^{-k}$ then 2 sets of diameter U cover $\{1/2^k, 1/2^{k+1}, 1/2^{k+2}, \dots\}$, leaving the $k-1$ points $\{1/2, 1/4, \dots 1/2^{k-1}\}$ to be covered by $k-1$ sets. Thus,

write as $[0, \frac{1}{2^k}]$

upper dimension here

$$\dim_B F \leq \lim_{k\to\infty} \frac{\log(k+1)}{-\log(2^{-k})} = \lim_{k\to\infty} \frac{\log(k+1)}{k \log 2} \checkmark$$

$$= 0 \checkmark \Rightarrow \dim_B F = 0 \checkmark$$

$\frac{6}{7}$

6) If $|U| = \delta < 1/2$ and k is the integer satisfying $1/k(k+1) \leq |U| < 1/k(k-1)$ then U can cover at most one point in the set $\{1, 1/2, 1/3, \dots 1/k\}$ thus at most k sets of diameter δ are needed to cover F, and

$$\dim_B F \geq \lim_{k\to\infty} \frac{\log k}{-\log(k^{-1}(k+1)^{-1})} = \lim_{k\to\infty} \left[\frac{\log k}{\log(k^2+k)}\right] = \frac{1}{2}$$

$$\geq \lim_{k\to\infty} \frac{\log k}{+\log(k-1)^2} ? = \lim_{k\to\infty} \frac{\log k}{2\log(k-1)} \quad \text{not quite}$$

$= 1/2$ ie ~~$\dim_B F$~~

P.T.O.

If $0 < \delta < 1/2$ take k such that $1/(k-1)k > \delta > 1/k(k+1)$ then $k+1$ sets of diameter δ cover