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b) i) $F \subset E_0 = \{(a,b) \in \mathbb{R}^2 : 0 \le a, b \le 1\}$ so E_0 is bounded and F is bounded.

Suppose we consider the stage k in the construction of F. At this stage the dust consists of the union of 4^k closed squares of diameter $4^{-k}\sqrt{2}$. If x and y are distinct points in the same square then $|x-y| > \delta > 0$ for some δ take k to be any number such that $\delta > \frac{\sqrt{2}}{4^k}$ ie $+\log\delta > \sqrt{2} - k\log 4$

$$k \geq \frac{\sqrt{2} - \log\delta}{\log 4} = \frac{\sqrt{2} - \log\delta}{2\log 2}$$

$\left(k \geqslant \frac{-\log\delta}{\log 2} \text{ will do}\right)$

Why do this here?

The x and y are not in the same square for this value of n since the distance between them is greater than the diameter of any square in E_n. Since x and y are arbitrary we can generalize and say F consists of 'dust particles' each of which contains at most one point, by taking the limit as $n \to \infty$. By a theorem of Cantor, for a decreasing sequence $\{E_n\}$ of non empty closed* subsets of a metric space M ($\mathbb{R}^2$ is complete ie every Cauchy sequence converges in $\mathbb{R}^2$) such that $E_{n+1} \subset E_n$ for all n and $\operatorname{diam}(E_n) \to 0$ as $n \to \infty$, $\bigcap_{n=1}^{\infty} E_n \neq \emptyset$.

Therefore each particle of dust

a big step

You need to state somewhere that E_k is closed since it is the union of finitely many closed sets.

* use compact here then F is a non-empty compact set. see problem 3, p 18 of course notes (in Falconer)