2)(a) $\underline{\lim}_{n\to\infty} a_n = \underline{\lim}_{N\to\infty} \inf_{n>N} a_n = 0$ ✓ (3)

Since $a_n > 0$ for all n and if $\varepsilon > 0$ we can find $a_n = \frac{1}{n} < \varepsilon$.

as $N \to \infty$ $\inf a_n \to 0$. $\inf_{n>N} a_n = 0$

$\overline{\lim}_{n\to\infty} a_n = \lim_{N\to\infty} \sup_{n>N} a_n = \infty$ ✓

Since given M we can find n such that $n = a_n > M$, for any such M

$\therefore$ as $N \to$ inf? $\sup a_n \to \infty$. $\sup_{n>N} a_n = \infty$

$\frac{2}{4}$

b) $-1 \leq \cos x \leq 1$

$\therefore \frac{-1}{x^2} \leq f(x) = \frac{\cos x}{x^2} \leq \frac{1}{x^2}$

As $x \to \infty$ $\lim_{K\to\infty} (\inf f(x)) \geq$

$\underline{\lim}_{x\to\infty} f(x) \geq \lim_{K\to\infty} \cancel{(-1/x^2)}_{\cancel{x>K}}\ {-\frac{1}{K^2}} = 0$

$\overline{\lim}_{x\to\infty} f(x) = \lim_{K\to\infty} \sup_{x>K} (f(x))$ ✓

$\leq \lim_{K\to\infty} \cancel{(1/x^2)}_{\cancel{x>K}}\ \frac{1}{K^2} = 0$

Hence $\underline{\lim}_{x\to\infty} f(x) = \overline{\lim}_{x\to\infty} f(x) = 0$

$\frac{2}{5}$

c) $-1 \leq f(x) \leq 1$ for all $x \in \mathbb{R}^+$

Since $f(x) \geq -1$ we know that $\inf(f(x)) \geq -1$ ✓

Notice that if $x_n = \frac{1}{(2n+1)n\pi}$ then $f(x) = -1$ ✓

Since x is irrational. For $\varepsilon^r > 0$ we can find n satisfying

not needed $0 < \frac{1}{(2n+1)n\pi} < \varepsilon^r$ Simply state here that for each $r > 0$, the interval $(0, r)$ contains an irrational number, so $\inf\{f(x) : 0 < x < r\} = -1$