

Please make sure that the assignment number is correctly entered on your PT3 form as

M835 01.

This assignment assesses the material covered in Chapters 1 and 2 of the Course Notes.

**Question 1** – 25 marks

- (a) Let  $A = \{(a, b) \in \mathbb{R}^2 : 0 \leq a < 1, 0 \leq b \leq 1\}$ .
- Determine whether  $A$  is open, closed or neither.
  - Write down  $\bar{A}$ ,  $\text{int } A$  and  $\partial A$ .
- (b) Let  $F$  be the Cantor dust shown on page xvi of Falconer, with  $E_0 = \{(a, b) \in \mathbb{R}^2 : 0 \leq a, b \leq 1\}$ . Show that  $F$  is
- compact;
  - perfect;
  - totally disconnected.

**Question 2** – 15 marks

- (a) Find  $\lim_{n \rightarrow \infty} a_n$  and  $\overline{\lim}_{n \rightarrow \infty} a_n$ , where
- $$a_n = \begin{cases} n, & \text{if } n \text{ is even,} \\ 1/n, & \text{if } n \text{ is odd.} \end{cases}$$
- (b) Find  $\lim_{x \rightarrow \infty} f(x)$  and  $\overline{\lim}_{x \rightarrow \infty} f(x)$ , where  $f: \mathbb{R}^+ \rightarrow \mathbb{R}$  is given by
- $$f(x) = \frac{\cos x}{x^2}.$$
- (c) Find  $\lim_{x \rightarrow 0} f(x)$  and  $\overline{\lim}_{x \rightarrow 0} f(x)$ , where  $f: \mathbb{R}^+ \rightarrow \mathbb{R}$  is given by
- $$f(x) = \begin{cases} \cos(1/x), & \text{if } x \text{ is rational,} \\ -1, & \text{if } x \text{ is irrational.} \end{cases}$$

**Question 3** – 20 marks

Let  $F$  consist of those real numbers in  $[0, 1]$  whose non-terminating decimal expansions contain only odd digits.

- Construct sets  $E_k$  such that  $F = \bigcap_{k=0}^{\infty} E_k$  and sketch the sets  $E_0$ ,  $E_1$  and  $E_2$ .
- Construct a measure supported on  $F$ .
- Find  $\dim_H F$ .

$$\mathcal{L}(U) = \mathcal{L}(U \cap F)$$