In an exam, if you don't use the stated methods you will get no marks

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the set of points $\{1/k, 1/(k+1), \ldots\}$ leaving $k-1$ points which can be covered by another $k-1$ sets ✓

thus $\dim_B G \leq \lim_{k\to\infty} \frac{\log(2k)}{\log(k(k-1))}$

$$\leq \lim_{k\to\infty} \frac{\log 2k}{\log(k-1)^2} = \lim_{k\to\infty} \frac{\log k + \log 2}{2\log k - 1}$$

$= \frac{1}{2}$ i.e. $\dim_B G = \frac{1}{2}$ ✓ * You were asked to use the result of example 3.5 of Falconer to find $\dim_B G$. $\dim_B\{G \cup \{0\}\} = \frac{1}{2}$

$\frac{1}{3}$ $\Rightarrow \dim_B F = \frac{1}{2}$

Also, $\dim_B(G \cup \{0\}) = \max\{\dim_B G, \dim_B\{0\}\}$. Since $\dim_B\{0\} = 0 \Rightarrow \dim_B G = \frac{1}{2}$

c) $|f(x) - f(y)| = |x^2 - y^2|$

$= |x-y||x+y|$

for $x, y \in [0,1]$, $|x+y| \leq 2$.

$\Rightarrow |f(x) - f(y)| \leq 2|x-y|$ ✓ for all x, y. $f(x)$ is a Lipschitz function

$x = y$ is not useful since $|f(x) - f(y)| = 0$ whenever $x = y$

P.T.O.

If f is bi-Lipschitz we also have $c|x-y| \leq |f(x) - f(y)|$, $x, y \in [0,1]$

for some $2 > c > 0$, then

$c|x-y| \leq |x-y||x+y|$ for all x, y +

if $x \neq y$ then ① $\Rightarrow |x+y| \geq c$ for all x, y

But this we cannot have. Take $x = 0$ and $y = \varepsilon < c$ where $\varepsilon > 0$, which we can do for any $c > 0$ then

$|x+y| = \varepsilon < c$, contradicting ① $\therefore$

$\frac{3}{4}$ f not bi-Lipschitz ✓

d) By Problem P9 of unit 2, if f satisfies a Hölder condition

$|f(x) - f(y)| \leq c|x-y|^\alpha$, $x, y \in F$, $\alpha \in (0,1]$

then $\dim_B f(F) \leq 1/\alpha \dim_B(F)$, ✓ State that for a Lipschitz function, $\alpha = 1$

$0 \leq \underline{\dim}_B f(F) \leq \overline{\dim}_B f(F) \leq \frac{\dim_B F}{\alpha} = 0$ ✓

$\dim_B f(F) = 0$ ✓

Since in this example, $\alpha = 1$.

You have to mention that F and ✓ G are contained in $[0,1]$