

(2)

then the square  $S$  is contained in  $B_S(x)$  as shown.

At the next stage  $S$  is subdivided into 4 squares, each contained in  $B_S(x)$  - if we take any point in one of the other

three squares, say  $y$ , then  $|x - y| < S$ . Now let  $k \rightarrow \infty$  then we can let  $S \rightarrow 0$  while

satisfying  $S \geq \sqrt{2}/4^k$  so for any  $S > 0$  we can find  $x, y \in F$  such that  $|x - y| < S$  and every open set containing  $x \in F$  contains some  $y \in F$  distinct from  $x \ni x \in F$  then  $x$  is a limit point of  $F$ .

iii) That  $F$  is totally disconnected was proved in i) where we showed that  $F$  is a union of distinct singleton sets. Not sufficient

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For each  $k \in \mathbb{N}$ , each component of  $E_k$  contains a point in  $F$ .

Thus if  $x \in F$ , then, for each  $k \in \mathbb{N}$ , there exists  $x_k \in F$  with  $|x_k - x| \leq \frac{1}{4^{k-1}}$ , so that  $x_k \rightarrow x$  as  $k \rightarrow \infty$ . Thus,  $x$  is a limit point of  $F$ .

+ If  $x$  and  $y$  are in the same component of  $E_k$  for each  $k \in \mathbb{N}$ , then  $|x - y| \leq \sqrt{2}/4^k$ , for each  $k \in \mathbb{N}$ , so  $x = y$ .

Thus, if  $x$  and  $y$  are distinct points in  $F$ , then there exists  $k \in \mathbb{N}$  such that  $x$  and  $y$  belong to different components  $E_{k,x}$  and  $E_{k,y}$  of  $E_k$ .

There exist disjoint open sets  $U$  and  $V$  such that

P.T.O.