

$E_k, x \in U$  while  $V$  contains the remaining  $4^k - 1$  squares in  $E_k$ , (including  $E_{k,y}$ ).

Thus,  $F \cap E_k \subset U \cup V$  while  $x \in U, y \in V$  and  $U \cap V = \emptyset$ .

So  $x$  and  $y$  are in different connected components of  $F$ , hence  $F$  is totally disconnected.