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for any $\varepsilon > 0$, a $\delta > 0$ such that $|\cos 1/x - \cos 2\pi n| < \varepsilon$ when $|x - 1/2\pi n| < \delta$

Hence $\lim_{x \to 0} f(x) = \lim_{k \to \infty} \left(\lim_{x \to k} f(x) \right) = 1$.

3) a) $E_0 = [0,1]$ ✓

We would have for the set E_1 only those intervals beginning 0.1, 0.3, ... 0.9, ending 0.2, 0.4, ...1 resp.

ie $E_1 = [0.1, 0.2] \cup [0.3, 0.4] \cup [0.5, 0.6] \cup [0.7, 0.8] \cup [0.9, 1]$.

Similarly for the set E_2 we would have only those intervals beginning 0.11, 0.13, 0.15 etc and ending 0.12, 0.14 (since $0.12 = 0.11\dot{9}\dot{9}$)

ie $E_2 = [0.11, 0.12] \cup [0.13, 0.14] \cup \dots \cup [0.99, 1]$. ✓ P.T.O.

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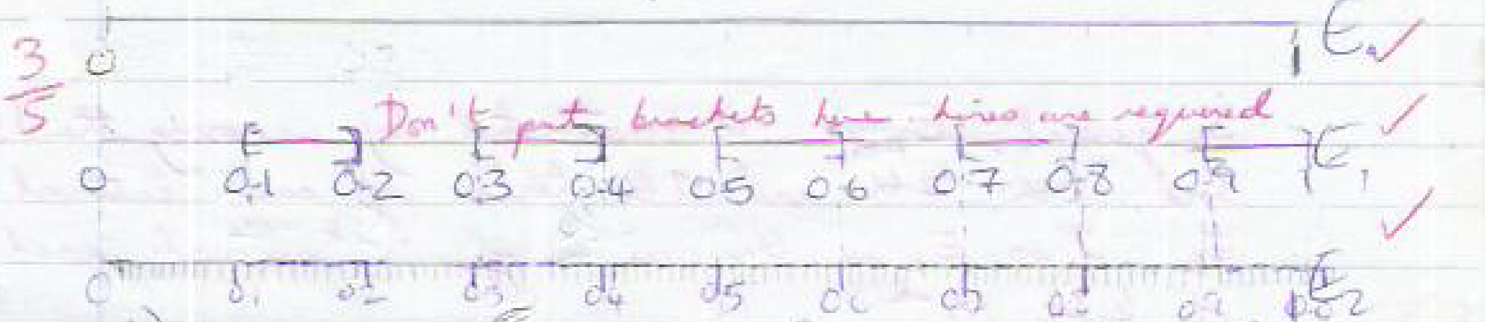

b) At the stage k of construction of F we have 5^k intervals of length 10^{-k}. } Part of (i)

If E_k is the union of those 5^k intervals → P.T.O.

then since is? such union is a subset of the previous such union ($E_k \subset E_{k-1}$), and $E_{k+1} \subset E_k$, and if $\{U_k\}$ is a decreasing sequence of sets, with $U_k \in E_k$ closed ~~and~~ compact, and non empty, $\bigcap_{k=1} U_k \neq \emptyset$ ✓

For each $k \in \mathbb{N}$ and each interval $U \in E_k$ $\mu(U) = 5^{-k}$ ✓ and $\mu(\mathbb{R} - \cancel{F}\, E_k) = 0$.