Question 4 – 20 marks

Let F be defined in the following way. Take E_0 to be the interval $[0,1]$, construct E_k by replacing the middle third of each straight-line segment of E_k by the other three sides of the square based on the removed segment and let F be the limit of the polygonal curves E_k.

(a) Sketch the curves E_0, E_1 and E_2.

(b) Determine the similarity dimension of F.

(c) By considering values of δ of the form $1/3^k$, where $k \in \mathbf{R}$, plot a log-log graph for F of the type shown in Figure 3.1 of Falconer, but with $-\log$ values on the horizontal axis. Use your graph to estimate the divider dimension of F.

(d) Use parts (iv) and (v) of Equivalent definitions 3.1 of Falconer to determine $\dim_B F$.

Question 5 – 20 marks

Let $F = \{1/2^n : n \in \mathbf{N}\}$, $G = \{1/n : n \in \mathbf{N}\}$ and $f : [0,1] \to [0,1]$ be the function defined by $f(x) = x^2$.

(a) Determine $\dim_B F$.

(b) Use the result of Example 3.5 of Falconer to determine $\overline{\dim}_B G$.

(c) Verify that f is a Lipschitz function but is not a bi-Lipschitz function.

(d) Hence determine $\dim_B f(F)$ and obtain an upper bound for $\overline{\dim}_B f(G)$.

TMA M835 02 — Cut-off date 23 May 2001

Please make sure that the assignment number is correctly entered on your PT3 form as

$\boxed{\text{M835 02}}$.

This assignment assesses the material covered in Chapters 3 and 4 of the Course Notes.

Question 1 – 15 marks

Let $F = \{(a,b) : 0 \le a, b \le 2\} \subset \mathbf{R}^2$. Show, from the definition, that $\mathcal{H}^s(F) = 0$ if $s > 2$, $0 < \mathcal{H}^2(F) < \infty$ and $\mathcal{H}^s(F) = \infty$ if $s < 2$. (You may need to use the fact that, if U is a subset of $\mathbf{R}^2$, then the area of U is at most equal to $|U|^2 \times \pi/4$ — the area of a disc of the same diameter.)

Question 2 – 20 marks

Let $f : \mathbf{R} \to \mathbf{R}$ be a differentiable function with a continuous derivative and let F be a subset of $\mathbf{R}$.

(a) Use your solutions to Problems ~~10~~ 12 and ~~13~~ 15 of Chapter 1 of the Course Notes to show that, if F is bounded, then $\dim_H f(F) \le \dim_H F$.

(b) Using the countable stability of Hausdorff dimension, deduce that $\dim_H f(F) \le \dim_H F$, for any subset F of $\mathbf{R}$.

(c) Give an example of a subset F of $\mathbf{R}$ and a differentiable function $f : \mathbf{R} \to \mathbf{R}$ with a continuous derivative such that $\dim_H f(F) < \dim_H F$.