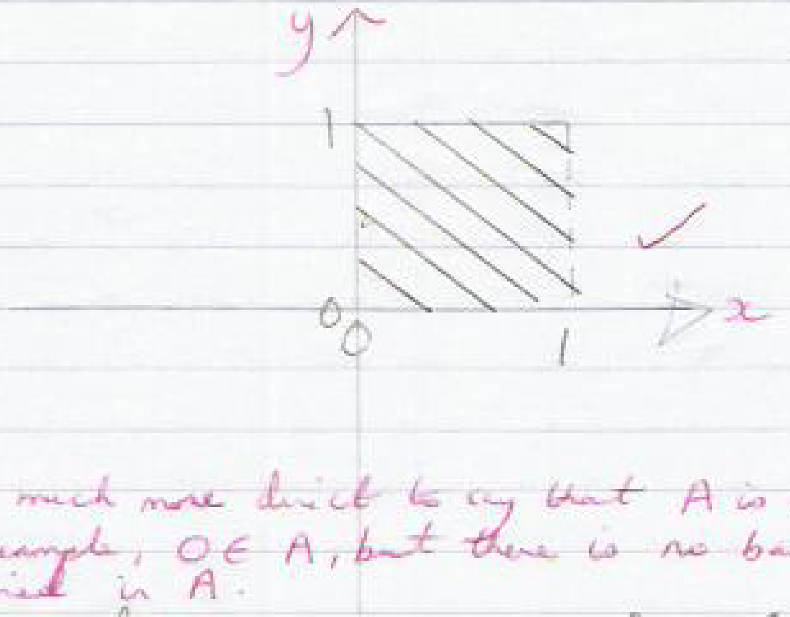
TMA M835 01 (1)

1)(a)



✓

It is much more direct to say that A is not open since, for example, $0 \in A$, but there is no ball $B_r(0)$ which is contained in A.

A is not open since if $x_n = (a, 1 + 1/n) \in \mathbb{R}^2 - A$, $a \in [0, 1)$ then the limit point of the sequence is $(a, 1)$. If $(a, 1) \in \mathbb{R}^2 - A$ is closed then A is open. Suppose then that $\mathbb{R}^2 - A$ is closed then every convergent sequence converges to a limit point in $\mathbb{R}^2 - A$ but x_n converges to $(a, 1)$ and $(a, 1) \notin \mathbb{R}^2 - A$ $\therefore$ $\mathbb{R}^2 - A$ not closed $\therefore$ A not open. ✓

Suppose A closed then A includes all its limit points. $y_n = \{1 - 1/n, b\}$ is a sequence with $y_n \in A$ for all n, $b \in [0, 1]$ but the limit point of $\{y_n\}$ is $(1, b)$ ✓ and $(1, b) \notin A$ $\therefore$ A not closed.

Alternatively observe that $\mathbb{R}^2 - A$ not open since $(1, a^*) \in \mathbb{R}^2 - A$ specify a
but every open ball $B_\varepsilon(1, a)$, with $\varepsilon > 0$
$\frac{5}{5}$ contains the point $(1 - \varepsilon/2, a) \in A$
$\mathbb{R}^2 - A$ not open $\Rightarrow$ A not closed.
So A is neither open nor closed. The same argument works to prove that A is not open ✓ as I have indicated above