P6578708 (6)

On $[i-\frac{1}{2}, i+\frac{1}{2}]$ $l_0(x)$ satisfies

$$l_0(i) = 0$$
$$l_0(i-\tfrac{1}{2}) = (2-\sqrt{2})(2\sqrt{2}-3)^{i-1}$$
$$l_0(i+\tfrac{1}{2}) = (2-\sqrt{2})(2\sqrt{2}-3)^{i}$$

from E 9.5 18.39 with $k = i-1$, p, $j=0$.

$$l_0(x) = 0 + (x-i)(1-(2\sqrt{2}-3)^{-1})(2-\sqrt{2})(2\sqrt{2}-3)^{i}$$
$$+ 2(x-i)^2(1+(2\sqrt{2}-3)^{-1})(2-\sqrt{2})(2\sqrt{2}-3)^{i}$$

$$= (x-i)(2-\sqrt{2})(2\sqrt{2}-3)^{i}(1+3+2\sqrt{2})$$
$$+ 2(x-i)^2(1-2\sqrt{2}-3)(2-\sqrt{2})(2\sqrt{2}-3)^{i}$$

$$= (x-i)(2-\sqrt{2})(2\sqrt{2}-3)^{i}(4+2\sqrt{2})$$
$$+ 2(x-i)^2(2-\sqrt{2})(2\sqrt{2}-3)^{i}(-2-2\sqrt{2}).$$

$$= (x-i)(2-\sqrt{2})(2\sqrt{2}-3)^{i}(4+2\sqrt{2}-(4+4\sqrt{2})(x-i))$$

$$= (x-i)(2-\sqrt{2})(2\sqrt{2}-3)^{i}((4+2\sqrt{2})(1-\sqrt{2}(x-i))$$

$$= (x-i)(2-\sqrt{2})(4+2\sqrt{2})(2\sqrt{2}-3)^{i}(1-\sqrt{2}(x-i))$$

$$= (x-i)(8+4\sqrt{2}-4\sqrt{2}-4)(2\sqrt{2}-3)^{i}(1-\sqrt{2}(x-i))$$

$$= 4(x-i)(2\sqrt{2}-3)^{i}(1-\sqrt{2}(x-i)).$$ ✓

ii) $$\sum_{j=-\infty}^{\infty} l_j(\tfrac{1}{4}) = \sum_{j=0}^{-\infty} l_j(\tfrac{1}{4}) + \sum_{j=1}^{\infty} l_j(\tfrac{1}{4}) = \sum_{j=0}^{\infty} l_{-j}(\tfrac{1}{4}) + \sum_{j=1}^{\infty} l_j(\tfrac{1}{4})$$

Now $l_{-1}(\tfrac{1}{4}) = l_0(\tfrac{5}{4})$

$l_{-2}(\tfrac{1}{4}) = l_0(2+\tfrac{1}{4})$

$\vdots$

$l_{-j}(\tfrac{1}{4}) = l_0(j+\tfrac{1}{4})$