

$$= \frac{-2^3}{24}$$

$$\sum_{i=0}^3 B_i^3(x) = \sum_{i=0}^3 B_{i+1}^3(x) \quad \text{since } B_{i+1}^k(x) = B_i^k(x)$$

$$\text{for } 0 \leq x \leq 1$$

$$= \frac{1}{24} (-2^3 + 3x^3 - 3x + 1 + 3x^3 - 6x^2 + 4 - 3x^3 + 3x^2 + 3x + 1 + x^3) = \frac{6}{24} = \frac{1}{4}$$

ii) On  $[0, 1]$  with  $\xi_j = i, i = 0, \pm 1, \pm 2, \dots$   
 the only non zero  $B_p$  are for  
 $p = 0, 1, 2, 3$   
 On  $[0, 1]$   $x = \sum_{p=0}^3 \lambda_p B_p$

Put  $x_0 = 0, x_1 = \frac{1}{3}, x_2 = \frac{2}{3}, x_3 = 1$  and  
 form the matrix  $\frac{1}{24} \sum_{p=0}^3 \lambda_p B_p^3(x_i) \quad i = 0, 1, 2, 3$

$$\begin{bmatrix} 1 & 4 & 1 & 0 & 0 \\ \frac{3}{27} & \frac{9}{27} & \frac{6}{27} & \frac{1}{27} & 0 \\ \frac{1}{27} & \frac{6}{27} & \frac{9}{27} & \frac{3}{27} & 0 \\ 0 & 1 & 4 & 1 & 24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 5 & 1 & 24 \\ \frac{9}{27} & \frac{15}{27} & \frac{15}{27} & \frac{9}{27} & 24 \\ \frac{7}{27} & \frac{13}{27} & \frac{13}{27} & \frac{7}{27} & 8 \\ -1 & -5 & 3 & +1 & 24 \end{bmatrix} \begin{array}{l} R_1 + R_4 \\ R_2 + R_3 \\ R_3 - R_2 \\ R_4 - R_1 \end{array}$$

$$\begin{bmatrix} 0 & -12 & -2 & 0 & -48 \\ \frac{1}{3} & \frac{17}{3} & \frac{17}{3} & \frac{1}{3} & 24 \\ 0 & \frac{12}{27} & \frac{12}{27} & 0 & \frac{16}{9} \\ -1 & -3 & 3 & 1 & 24 \end{bmatrix} \begin{array}{l} R_1 - 3R_2 \\ R_2 \times 3 \\ R_3 - \frac{7}{27}R_4 \end{array}$$