Question 2 (Chapter 18) – 20 marks

(i) Let $\ell_j(x)$, $j = 0, \pm 1, \pm 2, \ldots$, be the quadratic cardinal spline functions which satisfy equation (18.39) (Powell, page 223) with $h = 1$. Show that

$$\ell_0(x) = \begin{cases} 1 - 4(\sqrt{2} - 1)x^2, & -\frac{1}{2} \le x \le \frac{1}{2}, \\ 4(2\sqrt{2} - 3)^i (x - i)(1 - \sqrt{2}(x - i)), & i - \frac{1}{2} \le x \le i + \frac{1}{2},\ i = 1, 2, \ldots . \end{cases}$$

(ii) Use your answers to part (i) to verify that

$$\sum_{j=-\infty}^{\infty} \ell_j\left(\tfrac{1}{4}\right) = 1.$$

Question 3 (Chapter 19) – 25 marks

In this question $\xi_j = j$, $j = 0, \pm 1, \pm 2, \ldots$, and B_p^k, $p = 0, \pm 1, \pm 2, \ldots$, $k = 1, 2, \ldots$, denote the corresponding B-splines of degree k.

(i) Write down formulas for B_{-3}^3, B_{-2}^3, B_{-1}^3, B_0^3 on $[0, 1]$, and use these formulas to verify that

$$B_{j-3}^3(x) + B_{j-2}^3(x) + B_{j-1}^3(x) + B_j^3(x) = \tfrac{1}{4}, \qquad \xi_j \le x \le \xi_{j+1}.$$

(ii) Determine the coefficients λ_p, $p = 0, \pm 1, \pm 2, \ldots$, such that

$$x = \sum_{p=-\infty}^{\infty} \lambda_p B_p^3(x), \qquad x \in \mathcal{R}.$$

(*Hint*: first find λ_{-3}, λ_{-2}, λ_{-1}, λ_0, and then use part (i).)

(iii) Suppose that $n \ge 1$ and that

$$s(x) = \sum_{p=-3}^{n-1} \alpha_p B_p^3(x), \qquad 0 \le x \le n,$$

where $\alpha_p \in \mathcal{R}$, $p = -3, -2, \ldots, n-1$. Prove that

$$s'(x) = \tfrac{3}{4} \sum_{p=-2}^{n-1} (\alpha_p - \alpha_{p-1}) B_p^2(x), \qquad 0 \le x \le n.$$

Question 4 (Chapter 19) – 15 marks

Let $\xi_j = j$, $j = -2, -1, \ldots, 3$, and let $B_p(x)$, $p = -2, -1, 0$, denote the corresponding quadratic B-splines. Use Theorem 19.4 (Powell, page 237) to determine the coefficients λ_p such that (19.37) holds with $k = 2$ and the following data.

i	1	2	3
x_i	0	$\frac{1}{2}$	1
$f(x_i)$	1	2	3

Question 5 (Chapter 22) – 20 marks

(i) Evaluate $p(-1)$, where p is the quadratic polynomial which interpolates the data $\{f(0), f(1), f'\left(\frac{1}{4}\right)\}$.

(ii) Deduce that if $f \in \mathcal{C}^{(3)}[-1, 1]$, then

$$f(-1) + 2f(0) - 3f(1) + 4f'\left(\tfrac{1}{4}\right) = -\tfrac{13}{24} f^{(3)}(\xi),$$

for some point ξ in $[-1, 1]$.

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