P65787087 (7)

And $l_1(1/4) = l_0(-1+1/4)$

$l_2(1/4) = l_0(-2+1/4)$

$\vdots \qquad \vdots$

$l_j(1/4) = l_0(-j+1/4)$

So $\sum_{j=-\infty}^{\infty} l_j(1/4) = \sum_{j=0}^{\infty} l_0(j+1/4) + \sum_{j=1}^{\infty} l_0(-j+1/4) + l_0(1/4)$

l_0 is even about zero $\therefore l_0(-x) = l_0(x)$

$\Rightarrow \sum_{j=1}^{\infty} l_0(-j+1/4) = \sum_{j=1}^{\infty} l_0(j-1/4)$

$\therefore \sum_{j=-\infty}^{\infty} l_j(1/4) = \sum_{j=0}^{\infty} l_0(j+1/4) + \sum_{j=1}^{\infty} l_0(j-1/4) + l_0(1/4)$

$l_0(1/4) = 1 - 1/4(\sqrt{2}-1) = \frac{5-\sqrt{2}}{4}$

$l_0(j+1/4) = 4\alpha(1/4)(2\sqrt{2}-3)^j(1-\sqrt{2}/4)$

$= \frac{1}{4}(4-\sqrt{2})(2\sqrt{2}-3)^j$

$l_0(j-1/4) = 4(-1/4)(2\sqrt{2}-3)^j(1+\sqrt{2}/4)$

$= -1/4(4+\sqrt{2})(2\sqrt{2}-3)^j$

$\sum_{j=0}^{\infty} l_0(j+1/4) = \frac{1}{4}(4-\sqrt{2})\left(\frac{2\sqrt{2}-3}{1-(2\sqrt{2}-3)}\right)$

$= \frac{1}{4}(4-\sqrt{2})\left(\frac{2\sqrt{2}-3}{4-2\sqrt{2}}\right) \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}}$

$= \frac{(-\cancel{10}\,6+11\sqrt{2})(4+2\sqrt{2})}{32}$

$= \frac{(-20+12\sqrt{2})}{32} = \frac{3\sqrt{2}-5}{8}$

$\sum_{j=1}^{\infty} l_0(j-1/4) = \frac{-(4+\sqrt{2})}{4}\left(\frac{2\sqrt{2}-3}{1-(2\sqrt{2}-3)}\right)$

$= \frac{-(4+\sqrt{2})}{4}\left(\frac{2\sqrt{2}-3}{4-2\sqrt{2}}\right) \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}}$

$= -\frac{(4+\sqrt{2})}{4}\frac{(8\sqrt{2}+8-12-6\sqrt{2})}{(16-8)}$

$= -\frac{(4+\sqrt{2})(2\sqrt{2}-4)}{32}$