P65787708

3

$$c_3 = \frac{132/7 + 192/7}{6 \times 1/2} = \frac{108}{7}$$

$$S_2(x) = 1 - \frac{\cancel{96}x^2}{7} + \frac{108x^3}{7}$$

On $[1/2, 1]$ $c_1 = \frac{0 + 1/2}{1/2} - \frac{(1/2)(\cancel{2}64/7 + 0)}{6} = \frac{-15}{7}$

$$c_3 = \frac{-132/7}{6 \times 1/2} = \frac{-44}{7}$$

$$S_3(x) = \frac{-1}{2} + \frac{15(x - 1/2)}{7} + \frac{66(x-1/2)^2}{7} - \frac{44(x-1/2)^3}{7}$$

ii) The quadratic spline has 3 sections:

$S_0 = a_1 + b_1x + c_1x^2$ on $[-1/2, -1/4]$

$S_1 = a_2 + c_2x^2$ on $[-1/4, 1/4]$

$S_2 = a_1 - b_1x + c_1x^2$ on $[1/4, 1]$

Note the symmetry even about zero, implied by the evenness of the data.

Also $s_0(-1) = 0$ (1)

$s_0(-1/2) = -1/2$ (2)

$s_0(-1/4) = s_1(-1/4)$ (3)

$s_0'(-1/4) = s_1'(-1/4)$ (4)

$s_0(0) = 1$ (5)

(1) $\Rightarrow$ $a_1 - b_1 + c_1 = 0$

(2) $\Rightarrow$ $a_1 - 1/2 b_1 + 1/4 c_1 = -1/2$

(3) $\Rightarrow$ $a_1 - 1/4 b_1 + 1/16 c_1 = a_2 + 1/16 c_2$

(4) $\Rightarrow$ $b_1 - 1/2 c_1 = -1/2 c_2$

(5) $\Rightarrow$ $a_2 = 1$

we write the system thus

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & 0 & 0 \\ 1 & -1/2 & 1/4 & 0 & 0 & -1/2 \\ 1 & -1/4 & 1/16 & -1 & -1/16 & 0 \\ 0 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

a_1, b_1, c_1
a_2, c_2
from left.