PG5786708 (2)

$$\begin{bmatrix} 1 & 0 & -1/15 & 33/5 \\ 0 & 1 & 4/15 & -192/5 \\ 0 & 0 & 56/15 & 352/5 \end{bmatrix} \quad R1 - \tfrac{1}{4} R2$$

$R3 - R2$

$$\begin{bmatrix} 1 & 0 & -1/15 & 33/5 \\ 0 & 1 & 4/15 & -192/5 \\ 0 & 0 & 1 & 132/7 \end{bmatrix} \quad R3 \times 15/56$$

$$\begin{bmatrix} 1 & 0 & 0 & 132/7 \\ 0 & 1 & 0 & -192/7 \\ 0 & 0 & 1 & 132/7 \end{bmatrix} \quad \begin{matrix} R1 + 1/15\, R3 \\ R2 - 4/15\, R3 \end{matrix}$$

$S''(-1) = 0$ Since S a natural spline

$S''(-1/2) = 132/7$

$S''(0) = -192/7$

$S''(1/2) = 132/7$

$S''(1) = 0.$

We use

$$S_j(x) = f(x_j) + c_1(x - x_j) + \tfrac{1}{2} S''(x_j)(x - x_j)^2 + c_3(x - x_j)^3$$

$$c_1 = \frac{f(x_{j+1}) - f(x_j)}{x_{j+1} - x_j} - \frac{(x_{j+1} - x_j)(2S''(x_j) + S''(x_{j+1}))}{6}$$

$$c_3 = \frac{S''(x_{j+1}) - S''(x_j)}{6(x_{j+1} - x_j)}$$

On $[-1, -1/2]$ $c_1 = \dfrac{-1/2 - 0}{1/2} - \dfrac{1/2(2 \times 0 + 132/7)}{6} = -\dfrac{18}{7}$

$$c_3 = \frac{132/7 - 0}{6 \times 1/2} = \frac{44}{7}$$

$$S_0(x) = \frac{-18}{7}(x+1) + \frac{44}{7}(x+1)^3 \quad \checkmark$$

On $[-1/2, 0]$ $c_1 = \dfrac{1 - -1/2}{1/2} - \dfrac{1/2(264/7 - 192/7)}{6} = \dfrac{15}{7}$

$$c_3 = \frac{-192/7 - 132/7}{6 \times 1/2} = -\frac{108}{7}$$

$$S_1(x) = -1/2 + \frac{15}{7}(x + 1/2) - \frac{66}{7}(x + 1/2)^2 - \frac{108}{7}(x + 1/2)^3 \quad \checkmark$$

On $[0, 1/2]$ $c_1 = \dfrac{-1/2 - 1}{1/2} - \dfrac{1/2(-384/7 + 132/7)}{6} = 0$