P6578708 (12)

$$+(-4B_{j-3}(x)+0\cdot B_{j-2}(x)+4B_{j-1}(x)+8B_j(x))$$

$$=(4j-4)B_{j-3}(x)+4jB_{j-2}(x)+(4j+4)B_{j-1}(x)+(4j+8)B_j(x)$$

$$1=\sum_{p=j-3}^{j}(8+4p)B_p(x)\text{, as I speculated.}$$

iii) $$S(x)=\sum_{p=-3}^{n-1}\alpha_p B_p^3(x)$$

$$=\sum_{p=-3}^{n-1}\frac{\alpha_p}{24}\sum_{k=0}^{4}\frac{4!(-1)^k}{k!(4-k)!}(x-(p+k))_+^3$$

$$S'(x)=\sum_{p=-3}^{n-1}\frac{3\alpha_p}{24}\sum_{k=0}^{4}\frac{4!(-1)^k}{k!(4-k)!}(x-(p+k))_+^2$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\frac{\alpha_p}{6}\sum_{k=0}^{4}\frac{4!(-1)^k}{k!(4-k)!}(x-(p+k))_+^2$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\frac{\alpha_p}{6}\left((x-p)_+^2+(x-(p+4))_+^2+\sum_{k=1}^{3}\frac{4!(-1)^k}{k!(4-k)!}(x-(p+k))_+^2\right)$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\frac{\alpha_p}{6}\left((x-p)_+^2+(x-(p+4))_+^2+\sum_{k=1}^{3}\left(\frac{3!(-1)^k}{k!(3-k)!}+\frac{3!(-1)^k}{(k-1)!(3-(k-1))!}\right)(x-(p+k))_+^2\right)$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\frac{\alpha_p}{6}\left((x-p)_+^2+\sum_{k=1}^{3}\frac{3!(-1)^k}{k!(3-k)!}(x-(p+k))_+^2\right.$$
$$\left.+(x-(p+4))_+^2+\sum_{k=1}^{3}\frac{3!(-1)^k}{(k-1)!(3-(k-1))!}(x-(p+k))_+^2\right)$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\frac{\alpha_p}{6}\left(\sum_{k=0}^{3}\frac{3!(-1)^k(x-(p+k))_+^2}{k!(3-k)!}+(x-(p+4))_+^2+\sum_{k=1}^{3}\frac{3!(-1)^k(x-(p+k))_+^2}{(k-1)!(3-(k-1))!}\right)$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\alpha_p\left(B_p^2(x)+\frac{(x-(p+4))_+^2}{6}+\frac{1}{6}\sum_{k=0}^{3}\frac{3!(-1)^{k+1}}{k!(3-k)!}(x-(p+k+1))_+^2\right)$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\alpha_p\left(B_p^2(x)-\frac{1}{6}\sum_{k=0}^{3}\frac{3!(-1)^k}{k!(3-k)!}(x-(p+k+1))_+^2\right)$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\alpha_p\left(B_p^2(x)-B_{p+1}^2(x)\right).$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\alpha_p B_p^2(x)-\frac{3}{4}\sum_{p=-3}^{4}\alpha_p B_{p+1}^2(x)$$

$$=\frac{3}{4}\sum_{p=-3}^{n-1}\alpha_p B_p^2(x)-\frac{3}{4}\sum_{p=-2}^{5}\alpha_{p-1}B_p^2(x)$$ cont. opp