

$$= -\frac{(8\sqrt{2} - 16 + 4 - 4\sqrt{2})}{32} = -\frac{(4\sqrt{2} - 12)}{32}$$

$$= \frac{3 - \sqrt{2}}{8}$$

$$\sum_{j=0}^3 f_j\left(\frac{1}{4}\right) = \frac{5 - \sqrt{2}}{4} + \frac{3\sqrt{2} - 5}{8} + \frac{3 - \sqrt{2}}{8}$$

$$= \frac{5 - \sqrt{2}}{4} + \frac{2\sqrt{2} - 2}{8} = \frac{5 - \sqrt{2}}{4} + \frac{\sqrt{2} - 1}{4}$$

$$= \frac{4}{4} = 1 \text{ as required.} \checkmark$$

(2)

$$3) i) B_{-3}^3 = \frac{1}{24} ((x+3)_+^3 - 4(x+2)_+^3 + 6(x+1)_+^3 - 4(x)_+^3 + (x-1)_+^3)$$

$$= \frac{1}{24} ((x+3)^3 - 4(x+2)^3 + 6(x+1)^3 - 4x^3)$$

$$= \frac{1}{24} (x^3 + 9x^2 + 27x + 27 - 4x^3 - 24x^2 - 48x - 32)$$

$$+ 6x^3 + 18x^2 + 18x + 6 - 4x^3)$$

$$= \frac{1}{24} (-x^3 + 3x^2 - 3x + 1) \checkmark \text{ on } [0, 1]$$

$$B_{-2}^3 = \frac{1}{24} ((x+2)_+^3 - 4(x+1)_+^3 + 6(x)_+^3 - 4(x-1)_+^3 + (x-2)_+^3)$$

$$= \frac{1}{24} (x^3 + 6x^2 + 12x + 8 - 4x^3 - 12x^2 - 12x - 4 + 6x^3)$$

$$= \frac{1}{24} (3x^3 - 6x^2 + 4) \checkmark \text{ on } [0, 1]$$

$$B_{-1}^3 = \frac{1}{24} ((x+1)_+^3 - 4(x)_+^3 + 6(x-1)_+^3 - 4(x-2)_+^3 + (x-3)_+^3)$$

$$= \frac{1}{24} (x^3 + 3x^2 + 3x + 1 - 4x^3)$$

$$= \frac{1}{24} (-3x^3 + 3x^2 + 3x + 1) \checkmark \text{ on } [0, 1]$$

$$B_0^3 = \frac{1}{24} ((x)_+^3 - 4(x-1)_+^3 + 6(x-2)_+^3 - 4(x-3)_+^3 + (x-4)_+^3)$$